# ESSLLI08: Deontic Logic in Computer Science
# Part 3b/5: Norms, Obligations, Time and Agents

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## 1 Makinson's examples

David Makinson [5] illustrates the intricacies of temporal reasoning with norms, obligations and agents by discussing the iteration of detachment, in the sense that from the two conditional norms "if $p$, then obligatory $q$" and "if $q$, then obligatory $r$" together with the fact $p$, we can derive not only that $q$ is obligatory, but also that $r$ is obligatory. First, Makinson argues that iteration of detachment often appears to be appropriate by discussing the following example.

**Example 1 (Manuscript [5])** *Let the set of norms be "if $25x15$, then obligatory $text12$" and "if $text12$, then obligatory $refs10$", where $25x15$ is "The text area is 25 by 15 cm", $text12$ is "The font size for the main text is 12 points", and $refs10$ is "The font size for the list of references is 10 points". If the facts contain $25x15$, then we want to detach not only that it is obligatory that $text12$, but also that it is obligatory that $refs10$.*

Second, he argues that iteration of detachment sometimes appears to be inappropriate by discussing the following example, which he attributes to Sven Ove Hansson.

**Example 2 (Receipt [5])** *Let instances of the norms be "if $owe_{jp}$, then obligatory $pay_{jp}$" and "if $pay_{jp}$, then obligatory $receipt_{pj}$" where $owe_{jp}$ is "John owes Peter \$1000", $pay_{jp}$ is "John pays Peter \$1000", and $receipt_{pj}$ is "Peter gives John a receipt for \$1000". Intuitively Makinson would say that in the circumstance that John owes Peter \$1000, considered alone, Peter has no obligation to write any receipt. That obligation arises only when John fulfils his obligation.*

Makinson observes that there appear to be two principal sources of difficulty here. One concerns the passage of time, and the other concerns bearers of the obligations. Sven Ove Hansson's example above involves both of these factors. "We recall that our representation of norms abstracts entirely from the question of time. Evidently, this is a major limitation of scope, and leads to discrepancies with real-life examples, where there is almost always an implicit time element. This may be transitive, as when we say "when $b$ holds then $a$ should eventually hold", or "... should simultaneously hold". But it may be intransitive, as when we say "when $b$ holds then $a$ should hold within a short time" or "... should be treated as a matter of first priority to bring about". Clearly, iteration of detachment can be legitimate only when the implicit time element is either nil or transitive. Our representation also abstracts from the question of bearer, that is, who (if anyone) is assigned responsibility for carrying out what is required. This too can lead to discrepancies. Iteration of detachment becomes questionable as soon as some promulgations have different bearers from others, or some are impersonal (i.e. without bearer) while others are not. Only when the locus of responsibility is held constant can such an operation take place." [5]

## 2 Instantaneous (manuscript)

Broersen and van der Torre [3] introduce instantaneous semantics for the norms of the manuscript example, inspired by input/output logic.

**Definition 1 (Normative system)** *Let $L$ be a propositional language. A norm "if $i$, then obligatory $o$" is represented by a pair of formulas of $L$, and written as $(i, o)$. It is also read as the norm "if $i$, then forbidden $\neg o$." A normative system $S$ is a set of norms $\{(i_1, o_1), \ldots, (i_n, o_n)\}$.*

**Definition 2 (Temporal structure [3])** *A temporal structure is a tuple $T = \langle N, E, H \rangle$ where $N$ is a set of nodes, $E \subseteq N \times N$ is a set of edges obeying the properties of a tree, and $H : N \to 2^L$ is a function that associates with each node a maximal consistent subset of $L$ formulas holding at the node.*

**Definition 3 (Instantaneous semantics [3])** *The instantaneous semantics of a normative system $S$ is a function of temporal structures to obligation labelings $O : N \to 2^L$ such that for each node $n$, $O(n)$ is the unique minimal set such that:*

1. *for all norms $(i, o)$ and all nodes $n$, if $i \in Cn(H(n) \cup O(n))$, then $o \in O(n)$.*
2. *if $O(n) \models \varphi$ then $\varphi \in O(n)$, where $\models$ is logical consequence for propositional logic.*

**Definition 4 (Equivalence and redundancy)** *Two normative systems $S_1$ and $S_2$ are equivalent if and only if they have the same semantics, i.e., when they label each temporal structure in the same way. In normative system $S$, a norm $(i, o) \in S$ is redundant if and only if $S$ and $S \setminus \{(i, o)\}$ are equivalent.*

In this case, if a norm $s$ is redundant in normative system $S_1$, then it is also redundant in normative system $S_1 \cup S_2$. Consequently, two normative systems are equivalent if and only if each norm is redundant when added to the other normative system.

**Theorem 1 (Redundant instantaneous [3])** *In a normative system $S$, a norm $(i, o) \in S$ is redundant under the instantaneous semantics when we can derive it from $S \setminus \{(i, o)\}$ using replacement of logical equivalents in input and output, together with the following rules:*

$$\frac{}{(\bot, \bot)}\bot \quad \frac{}{(\top, \top)}\top \quad \frac{(i_1, o)}{(i_1 \wedge i_2, o)}SI \quad \frac{(i, o_1 \wedge o_2)}{(i, o_1)}WO$$

$$\frac{(i, o_1)(i, o_2)}{(i, o_1 \wedge o_2)}AND \quad \frac{(i_1, o)(i_2, o)}{(i_1 \vee i_2, o)}OR \quad \frac{(i, o_1), (i \wedge o_1, o_2)}{(i, o_2)}CT$$

When we replace $i \in Cn(H(n) \cup O(n))$ by $i \in H(n) \cup O(n)$ in the semantics, then we have to replace cumulative transitivity ($CT$) by transitivity ($T$) in the proof theory (note that without identity, $CT$ is not the same as $T$).

## 3 Persistent (receipt)

**Definition 5 (Persistent semantics [3])** *The persistent semantics of a normative system $S$ is the mapping of temporal structures to obligation labelings $O : N \to 2^L$ such that for each node $n$, $O(n)$ is the unique minimal set such that:*

1. *for all norms $(i, o)$, all nodes $n_1$ and all paths $(n_1, n_2, \ldots, n_m)$ with $m \geq 1$, if $i \in H(n_1)$ and $o \notin H(n_k)$ for $1 \leq k \leq m-1$, then $o \in O(n_m)$*
2. *if $O(n) \models \varphi$ then $\varphi \in O(n)$*

Compared to the instantaneous semantics, reasoning by cases is still supported, in the sense that the disjunction rule is still in force. However, we loose not only transitivity, but also the conjunction rule. E.g. requiring to take two pills together is not the same as requiring to take each of them, as in the latter case they may be taken at distinct time points.

**Theorem 2 (Redundant persistent [3])** *In a normative system $S$, a norm $(i, o) \in S$ is redundant under the persistence semantics when we can derive it from $S \setminus \{(i, o)\}$ using replacement of logical equivalents in input and output, together with $\bot$, $\top$, $SI$, $WO$ and $OR$.*

## 4 Deadline

**Definition 6 (Deadline norms)** *A norm "if $i$, then obligatory $o$ before $d$" is represented by a triple of formulas of $L$, and written as $(i, o, d)$.*

**Definition 7 (Deadline semantics [1])** *The deadline semantics of a normative system $S$ is the mapping of temporal structures to obligation labelings $O : N \to 2^L$ such that for each node $n$, $O(n)$ is the unique minimal set such that:*

1. *for all norms $(i, o, d)$, all nodes $n_1$ and all paths $(n_1, n_2, \ldots, n_m)$ with $m \geq 1$, if $i \in H(n_1)$ and $o \vee d \notin H(n_k)$ for $1 \leq k \leq m-1$, then $o \in O(n_m)$.*
2. *if $O(n) \models \varphi$ then $\varphi \in O(n)$*

**Theorem 3 (Redundant deadline [1])** *In a normative system $S$, a norm $(i, o, d) \in S$ is redundant under the deadline semantics when we can derive it from $S \setminus \{(i, o, d)\}$ using replacement of logical equivalents in input, output and deadline, together with the following rules:*

$$\frac{}{(\bot, \bot, \top)}\bot \quad \frac{}{(\top, \top, \top)}\top \quad \frac{(i_1, o, d)}{(i_1 \wedge i_2, o, d)}SI \quad \frac{(i, o_1 \wedge o_2, d)}{(i, o_1, d)}WO$$

$$\frac{(i_1, o, d)(i_2, o, d)}{(i_1 \vee i_2, o, d)}ORI \quad \frac{(i, o, d_1 \wedge d_2)}{(i, o, d_1)}WD \quad \frac{(i, o, d)}{(i, o, d \wedge o)}OSD$$

$$\frac{(i, o_1, \top), (i, o_2, \top)}{(i, o_1 \wedge o_2, i)}AND_\top \quad \frac{(i, o, p)(p, o, q)}{(i, o, q)}TRD$$

## 5 Violation

**Definition 8 (Violation semantics [1])** *The violation semantics of a normative system $S$ is the mapping of temporal structures to violation sets $V \subseteq N$ such that $n_m \in V$ if and only if there is a norm $(i, o, d)$, a node $n_1$ and a path $(n_1, n_2, \ldots, n_m)$ with $m \geq 1$, such that $i \in H(n_1)$ and $o \vee d \notin H(n_k)$ for $1 \leq k \leq m-1$, and $\neg o \wedge d \in H(n_m)$.*

**Theorem 4 (Violation redundancy [1])** *In a normative system $S$, a regulative norm $(i, o, d) \in S$ is redundant under the violation semantics if we can derive it from $S \setminus \{(i, o, d))\}$ using replacement of logical equivalents in input, output and deadline, together with the following rules:*

$$\frac{}{(\bot, \bot, \top)}\bot \quad \frac{}{(i, o, d \wedge o)}OD \quad \frac{(i_1, o, d)}{(i_1 \wedge i_2, o, d)}SI \quad \frac{(i, o_1 \wedge o_2, d)}{(i, o_1, d)}WO$$

$$\frac{(i_1 \wedge i_2 \wedge i_3, o, i_1 \wedge i_2)}{(i_1 \wedge i_2 \wedge i_3, o, i_1)}RWD \quad \frac{(i_1 \wedge i_2 \wedge i_3, o, i_1)}{(i_1 \wedge i_2 \wedge i_3, o, i_1 \wedge i_2)}RSD$$

$$\frac{(i_1, o, d)(i_2, o, d)}{(i_1 \vee i_2, o, d)}OR \quad \frac{(i, o_1, \top)(i, o_2, \top)}{(i, o_1 \wedge o_2, i)}AND_\top \quad \frac{(i, o, d_1)(i, o, d_2)}{(i, o, d_1 \vee d_2)}ORD$$

Moreover, if we care only about violations on traces, not where on the trace they occur, then other rules have to be added [1].

## 6 Beliefs and subjective

**Definition 9 (Epistemic temporal structure [3])** *Let $A$ be a set of agents. An epistemic temporal structure is a tree $\langle N, E, B \rangle$ where $N$ is a set of nodes, $E \subseteq N \times N$ is a set of edges obeying the properties of a tree, and $B : A \times N \to 2^L$ is a partial function such that $B(a, n)$ contains at least the tautologies and is deductively closed in $L$.*

**Definition 10 (Persistent subjective semantics [3])** *The persistent subjective semantics of a normative system $S$ is the mapping from epistemic temporal structures to agent obligation labeling $O : A \times N \to 2^L$ such that for each node $n$, $O(a, n)$ is the unique minimal set such that:*

1. *for all norms $(i, o)$, all nodes $n_1$ and all paths $(n_1, n_2, \ldots, n_m)$ with $m \geq 1$, if $i \in B(a, n_1)$ and $o \notin B(a, n_k)$ for $1 \leq k \leq m-1$, then $o \in O(a, n_m)$*
2. *if $O(a, n) \models \varphi$ then $\varphi \in O(a, n)$*

**Theorem 5 (Redundant persistent subjective [3])** *In a normative system $S$, a norm $(i, o) \in S$ is redundant under the subjective semantics when we can derive it from $S \setminus \{(i, o)\}$ using replacement of logical equivalents in input and output, together with $\bot$, $\top$, $SI$, $WO$.*

Along these lines, the normative system can be extended with permissive norms [2], constitutive norms like counts-as conditionals [1], hierarchies of normative systems with multiple authorities [2], and more. Dyadic obligations have been introduced [4] to model the situation where obligations might be fulfilled before they are detached.

## REFERENCES


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