ESSLLI08: Deontic Logic in Computer Science
Part 4b/5: Multiagent Games with Permissive Norms

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1 Weak and strong permission

Von Wright founded deontic logic in 1951 by observing that the relation between obligation and permission is like the relation between necessity and possibility, expressed by $Pp = \neg O \neg p$ and $\diamond p = \neg \Box \neg p$ respectively. Soon he realizes that there is not a single notion of permission, and he distinguishes *weak* from *strong* permission.

> "An act will be said to be permitted in the weak sense if it is not forbidden; and it will be said to be permitted in the strong sense if it is not forbidden but subject to norm. Strong permission only is a norm-character." [10, p.86].

Legal philosopher Bobbio [3, 4] relates permissions to the abolishing and derogating of norms, which is an important issue in legal theory. (Note that norm derogation also had important consequences for logic, because it led Alchourrón and Makinson to study contraction operators, which developed into the AGM paradigm in belief revision when they were joined by Gärdenfors.)

> "The difference between weak and strong permission becomes clear when we think about the function of permissive norms. Permissive norms are subsidiary norms: subsidiary in that their existence presupposes the existence of imperative norms [...] A permissive norm is necessary when we have to repeal a preceding imperative norm or to derogate to it. That is to abolish a part of it (that in this case it is not necessary preexisting because a law itself may prescribe a limit to its own extension)", [4, p. 891-892].
> "The function of permissive norms is to abolish an imperative in certain circumstances or with reference to certain persons [...] permissive norms are distinguished according to the fact that they abolish an *imperative which precedes in time* and in this case are called *abolishing norms*, or a *contemporary imperative*, and in this case they generally have the function of *derogating norms*." [3]

2 Games in hierarchical normative systems

Lewis [8] introduces in 1979 a game theoretic perspective on normative systems, by introducing a game consisting of two players, a master and a slave. In this game, obligations divide the possible actions of the slave into the sphere of prohibited actions and the sphere of permitted (i.e., not forbidden) actions, called "the sphere of permissibility". In this game, there is no need for strong permissions, weak permissions are sufficient.

Bulygin [7], who together with Alchourrón wrote an influential book on normative systems in 1971 [2], extends in 1986 Lewis games to hierarchical normative systems. As was already observed by Alchourrón and Makinson [1] and others before, when we consider the regulations in legal or administrative code, we can often discern some kind of hierarchy among them. Some are regarded as more basic or fundamental than others. They therefore study conflicts and *meta-norms*. The meta-norms of the normative system ascribe to each level of authority an area of competence (a set of propositions they can permit or forbid) and prescribe that the system must respect normative principles like "lex superior derogat inferiori" ("norms have the function of preventing - inhibit, preclude - the creation of imperative norms by subordinated sources of law"), "lex posterior derogat priori" (the function of abrogating preexisting imperative norms or to derogate to them), *etc.*

Bulygin [7] devices a new game, showing the role of permissions which do not refer to a preexisting prohibition:

> "Suppose that Rex, tired of governing alone, decides one day to appoint a Minister and to endow him with legislative power. [...] an action commanded by Minister becomes as obligatory as if it would have been commanded by Rex. But Minister has no competence to alter the commands and permissions given by Rex." If Rex permits hunting on Saturday and then Minister prohibits it for the whole week, its prohibition on Saturday remains with no effect." [7]

The game illustrates that "the role played by permissive norms is not exhausted by derogation of former prohibition: an act of permitting an action which has not been hitherto prohibited is not at all pointless as has been suggested by those who deny the importance of permissive norms" [7, p.213]. A normative system is composed of many authorities which are linked by hierarchical relations, as the "Rex, Minister and Subject" game shows, and a normative system has a dynamic character: norms are added to the system one after the other and this operation is performed by different authorities at different levels of the hierarchy.

(Note that there is a recent interest in similar games in computer science, where new logics for agents speaking for other agents are being developed in computer security, and the game theoretic perspective is gaining popularity in specification and verification.)

3 Permissions under constraints

The three notions of permission defined by Makinson and van der Torre [9] do not cover permissions as exceptions, while, as we have seen above, this is the principal role of permissive norms in legal systems. Most exceptions in the criminal code can be understood as such permissions, e.g., consider "it is forbidden to kill ($(\top, \neg k) \in G$), but it is permitted to kill in self-defense ($(s, k) \in P$), unless a policeman is killed ($(s \wedge p, \neg k) \in G$)". In input/output logic with constraints, these norms still imply the prohibition to kill in case of self-defense ($(s, \neg k) \in$ $out_{\cup/\cap}(G)$), because *maxfamily* and *outfamily* do not take permissions into account.

4 Formalization

4.1 Priorities

Boella and van der Torre [5, 6] introduce the following extension to permission in input/output logic.

Definition 1 (Permissions as exceptions) *Let G and P be disjoint sets of generators pointers, V a function that associates with every generator pointer a generator, and $\leq$ a partial pre-order on the powerset of $G \cup P$ that contains the subset-ordering. We read $A \leq B$ as B is at least as preferred as A.*

- *maxfamily(G, P, V, a) is the set $\subseteq$-maximal $G' \cup P'$ such that $G' \subseteq G$, $P' \subseteq P$ and out$(V(G') \cup V(Q), a) \cup \{a\}$ is consistent for every singleton or empty $Q \subseteq P'$.*
- *preffamily$(G, P, V, \leq, a)$ is the set of $\leq$ maximal elements of maxfamily(G, P, V, a).*
- *outfamily$(G, P, V, \leq, a)$ is the set of outputs of preffamily, i.e., the sets out$(V(G'), a)$ such that $G' \cup P' \in$ preffamily$(G, P, V, \leq, a)$, $G' \subseteq G$, and $P' \subseteq P$.*
- *statpermfamily$(G, P, V, \leq, a)$ is the set of out$(V(G' \cup Q), a)$ such that $G' \cup P' \in$ preffamily$(G, P, V, \leq, a)$, $G' \subseteq G$, $Q \subseteq P' \subseteq P$, and Q is a singleton or empty.*

The proof theory of this new kind of permission has not been studied yet.

4.2 Hierarchy of norms

Alchourrón and Makinson [1] define a hierarchy of regulations in this way: "a *hierarchy of regulations* to be a pair $(A, \leq)$ where A is a non-empty set of propositions, called a *code*, and $\leq$ is a partial ordering of A", p.126. Moreover, "[the judge] need[s] to compare, whenever possible, one set of regulations with another. In other words, given a relation $\leq$ that partially orders A, we need to envisage ways which $\leq$ induces some kind of ordering of 2^A", p.127.

Definition 2 (Hierarchy of norms) *A hierarchy is a partial pre-order $\preceq$ on generator pointers. A priority ordering on set of rules $\leq$ respects $\preceq$ when $B \leq C$ if for every $b \in B \setminus C$ there is a $c \in C \setminus B$ with $b \preceq c$. We write $a \prec b$ for $a \preceq b$ and $b \not\preceq a$.*

4.3 Redundant norms

Boella and van der Torre say that a norm is weakly redundant, when the output of a given set of norms does not change when we remove the norm.

Definition 3 (Static norms) *$g \in G \cup P$ is weakly redundant iff $\forall a \in L$: outfamily$(G, P, V, \leq, a) =$ outfamily$(G \setminus \{g\}, P \setminus \{g\}, V, \leq, a)$.*

A norm is strongly redundant when it is weakly redundant for any extension of the set of norms. The new norms may have any priority, and the priority relation among the old norms remains unchanged.

Definition 4 (Dynamic norms) *If $G \cup P$ is a set of norms extended with $G' \cup P'$, then we say that $\leq'$ extends $\leq$ if $\leq \subseteq \leq'$ and for all $g_1 \leq' g_2$ without $g_1 \leq g_2$, we have $g_1 \in G' \cup P'$ or $g_2 \in G' \cup P'$. The norm $g \in G \cup P$ is strongly redundant if and only if $\forall a \in L$, $\forall$ G', P' and $\leq'$ extending $\leq$, we have the same outputs, outfamily$(G \cup G', P \cup P', V, \leq', a)$ $=$ outfamily$(G \setminus \{g\} \cup G', P \setminus \{g\} \cup P', V, \leq', a)$.*

Intuitively, an authority may introduce a weakly but not strongly redundant norm to block the possibility that lower level authorities introduce conflicting and materially valid norms. Roughly, norms are strongly redundant when they are logically implied, i.e., derived by the input/output logic.

4.4 Competence

To model the scenarios of Bulygin and Lewis, Boella and van der Torre further detail the model of hierarchical normative systems by making the authorities and their competence explicit. In such a setting, they say that the lower and higher levels of authorities play a game against each other.

In the Bulygin/Lewis games, the hierarchy on rules is due to the "lex superior" principle. Other principles are discussed in the legal literature. These principles play the roles of *meta-norms* which "establish which norms do constitute a given legal order", i.e., in our terminology, meta-norms establish which norms are materially valid.

Definition 5 (Competence and formal validity) *Let A be a set of authorities, and $\succeq_A$ an ordering on A, $aut : G \cup P \rightarrow A$ a function that associates an authority with each rule, and $C : A \rightarrow 2^L$ the competence of authority expressed by a set of propositional formulas of L. We say that:*

- *The hierarchy $\preceq$ reflects $\preceq_A$ if and only if $aut(g_1) \prec_A aut(g_2)$ implies $g_1 \prec g_2$.*
- *The normative system respects the competence of the authorities if and only if for each norm $g = (x, y) \in G \cup P$ we have that $y \in C(aut(g))$.*
- *$g \in G \cup P$ is strongly redundant with respect to $\langle A, aut, \preceq_A, C\rangle$ if and only if it is strongly redundant for all normative systems respecting the competence of the authorities.*

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