What is a Joint Goal?
Games with Beliefs and Defeasible Desires

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Abstract

In this paper we introduce a qualitative decision and game theory based on belief (B) and desire (D) rules. We show that a group of agents acts as if it is maximizing achieved joint goals.

1 Introduction

One of the main problems in agent theory is the distinction between formal theories and tools developed for individual autonomous agents, and theories and tools developed for multi agent systems. In the social sciences, this distinction is called the micro-macro dichotomy. The prototypical example is the distinction between classical decision theory based on the expected utility paradigm (usually identified with the work of Neumann and Morgenstern [15] and Savage [11]) and classical game theory (such as the work of Nash and more recently the work of Axelrod). Whereas classical decision theory is a kind of optimization problem (maximizing the agent's expected utility), classical game theory is a kind of equilibria analysis.

There are several approaches in practical reasoning (within philosophy), cognitive science and artificial intelligence to bring the micro and macro description together. The basic idea is two-fold:

1. The decision making of individual autonomous agents is described in terms of other concepts than maximizing utility. For example, since the early 40s there is a distinction between classical decision theory and artificial intelligence based on utility aspiration levels and goal based planning (as pioneered by Simon [12]). Cognitive theories are typically based on vague concepts from folk psychology like beliefs, desires and intentions.
2. The decision making of a group or society of agents is described in terms of concepts generalized from those used for individual agents, such as joint goals, joint intentions, joint commitments, etc. Moreover, also new concepts are introduced at this social level, such as norms (a central concept in most social theories).

It is still an open problem how the micro-macro dichotomy of classical decision and game theory is related to the micro-macro dichotomy of these alternative theories. Has or can the micro and macro level be brought together by replacing classical theories by alternative theories? Before this question can be answered, the relation between the classical and alternative theories has to be clarified. Doyle and Thomason [6] argue that classical decision theory should be reunited with alternative decision theories in so-called qualitative decision theory (QDT), which studies qualitative versions of classical decision theory, hybrid combinations of quantitative and qualitative approaches to decision making, and decision making in the context of artificial intelligence applications such as planning, learning and collaboration. Qualitative decision theories have been developed based on beliefs (probabilities) and desires (utilities) using formal tools such as modal logic [2] and on utility functions and knowledge [7]. More recently these beliefs-desires models have been extended with intentions or BDI models [5, 10].

In this paper we introduce a rule based qualitative decision and game theory, based on belief (B) and desire (D) rules. We call an individual autonomous agent which minimizes its unreached desires a BD rational agent. We define goals as a set of formulas which can be derived by beliefs and desires in a certain way, such that BD rational agents act as if they maximize the set of achieved goals, and agents maximizing their sets of achieved goals are BD rational. Moreover, groups of agents which end up in equilibria act as if they maximize joint goals.

Like classical decision theory but in contrast to several proposals in the BDI approach [5, 10], the theory does *not* incorporate decision processes, temporal reasoning, and scheduling.

1.1 QDT and NMR

Qualitative decision theory is related to non-monotonic logic ("qualitative") and to reasoning about uncertainty ("decision theory" formalizes decision making under uncertainty). However, they are based on different disciplines. According to a distinction made by Aristotle, non-monotonic reasoning and reasoning about uncertainty formalize theoretical (or conclusion oriented) reasoning, whereas qualitative decision theory formalizes practical (or action oriented) reasoning.

Thomason [13] observes that classical decision theory neglects the issue of 'correct inference', and that the absence of a logical theory of practical reasoning is largely due to the unavailability of appropriate inference mechanisms. To handle even the simplest cases of practical reasoning, it is essential to have a reasoning mechanism that allows for practical conclusions that are non-monotonic in the agent's beliefs.

Where classical decision theory is based on probabilities and utilities, qualitative decision theory is based on beliefs and desires. In a modal approach, where the beliefs and desires are represented by modalities B and D respectively, and an action operator by the modal operator Do, we may have for example that:

$$B(thirsty), D(drink) \vdash Do(go-to-pub)$$

$$B(thirsty), D(drink), B(pub{-}closed) \vdash Do(go{-}to{-}shop)$$

A drawback of such a modal logic approach is that on the one hand modal logic is notorious for its problems with formalizing conditionals or rules, and practical reasoning on the other hand is usually seen as a kind of rule based reasoning. In this paper we therefore do not use modal logic but we use a rule based formalism.

The layout of this paper is as follows. Section 2 develops a qualitative logic of decision. This logic tells us what the optimal decision is, but it does not tell us how to find this optimal decision. Section 3 considers the AI solution to this problem [12, 8]: break down the decision problem into goal generation and goal based decisions.

2 A qualitative decision and game theory

The qualitative decision and game theory introduced in this section is based on sets of belief and desire rules. We define an agent system specification, we show how we can derive a game specification from it, and we give some familiar notions from game theory such as Pareto efficient decisions (choosing an optimal decision) and Nash equilibria. First we consider the logic of rules we adopt.

2.1 Logic of rules

The starting point of any theory of decision is a distinction between choices made by the decision maker and choices imposed on it by its environment. For example, a software upgrade agent (decision maker) may have the choice to upgrade a computer system at a particular time of the day. The software company (environment) may in turn allow/disallow such a upgrade at a particular time. Let $S = \{\alpha_1, \ldots, \alpha_n\}$ be the society or set of agents, then we therefore assume n disjoint sets of propositional atoms: $A = A_1 \cup \ldots \cup A_n = \{a, b, c, \ldots\}$ (agents' decision variables [7] or controllable propositions [2]) and $W = \{p, q, r, \ldots\}$ (the world parameters or uncontrollable propositions). In the sequel we consider each decision maker as entities consisting of defeasible rules. Such a decision maker generates its decisions by applying subsets of defeasible rules to its input. This results in the so-called conditional mental attitudes [4]. Before we proceed some notations will be introduced.

- L_{A_i}, L_W and L_{AW} for the propositional languages built up from these atoms in the usual way, and variables x, y, ... to stand for any sentences of these languages.
- Cn_{A_i}, Cn_W and Cn_{AW} for the consequence sets, and $\models_{A_i}$, $\models_W$ and $\models_{AW}$ for satisfiability, in any of these propositional logics.
- $x \Rightarrow y$ for an ordered pair of propositional sentences called a rule.
- $E_R(T)$ for the R extension of T, as defined in Definition 1 below.

In our framework the generation of decisions are formalized based on the notion of extension. In particular, the decision of an agent, which is specified by a set of defeasible rules R and has the input T, is the extension calculated based on R and T. This is formalized in the following definition.

Definition 1 (Extension) *Let $R \subseteq L_{AW} \times L_{AW}$ be a set of rules and $T \subseteq L_{AW}$ be a set of sentences. The consequents of the T-applicable rules are:*

$$R(T) = \{y \mid x \Rightarrow y \in R, x \in T\}$$

and the R extension of T is the set of the consequents of the iteratively T-applicable rules:

$$E_R(T) = \cap_{T \subseteq X, R(Cn_{AW}(X)) \subseteq X} X$$

We give some properties of the R extension of T in Definition 1. First note that $E_R(T)$ is *not* closed under logical consequence. The following proposition shows that $E_R(T)$ is the smallest superset of T closed under the rules R interpreted as inference rules.

Proposition 1 *Let*

- $E_R^0(T) = T$
- $E_R^i(T) = E_R^{i-1}(T) \cup R(Cn_{AW}(E_R^{i-1}(T)))$ *for* $i > 0$

We have $E_R(T) = \cup_0^\infty E_R^i(T)$.

Usually, an decision making agent is required to preserve its decisions under the growth of inputs. The following proposition shows that $E_R(T)$ is monotonic.

Proposition 2 *We have* $R(T) \subseteq R(T \cup T')$ *and* $E_R(T) \subseteq E_R(T \cup T')$.

Monotonicity is illustrated by the following example.

Example 1 *Let* $R = \{\top \Rightarrow p, a \Rightarrow \neg p\}$ *and* $T = \{a\}$, *where* $\top$ *stands for any tautology like* $p \vee \neg p$. *We have* $E_R(\emptyset) = \{p\}$ *and* $E_R(T) = \{a, p, \neg p\}$, *i.e. the* R *extension of* T *is inconsistent.*

Of course, allowing inconsistent decisions may not be intuitive. We are here concerned about possible decisions rather than reasonable or feasible decisions. Later we will define reasonable or feasible decisions by excluding inconsistent decisions.

2.2 Agent system specification

An agent system specification given in Definition 2 contains a set of agents and for each agent a description of its decision problem. The agent's decision problem is defined in terms of its beliefs and desires, which are considered as defeasible belief and desire rules, a priority ordering on the desire rules, as well as a set of facts and an initial decision (or prior intentions). We assume that agents are autonomous, in the sense that there are no priorities between desires of distinct agents.

Definition 2 (Agent system specification) *An agent system specification is a tuple* $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ *that contains a set of agents* S, *and for each agent* i *a finite set of facts* $F_i \subseteq L_W$ ($F = \bigcup_{i=1}^n F_i$), *a finite set of belief rules* $B_i \subseteq L_{AW} \times L_W$ ($B = \bigcup_{i=1}^n B_i$), *a finite set of desire rules* $D_i \subseteq L_{AW} \times L_{AW}$ ($D = \bigcup_{i=1}^n D_i$), *a relation* $\geq_i \subseteq D_i \times D_i$ ($\geq = \bigcup_{i=1}^n \geq_i$) *which is a total ordering (i.e. reflexive, transitive, and antisymmetric and for any two elements* d_1 *and* d_2 *in* D_i, *either* $d_1 \geq_i d_2$ *or* $d_2 \geq_i d_1$), *and a finite initial decision* $\delta_i^0 \subseteq L_A$ ($\delta^0 = \bigcup_{i=1}^n \delta_i^0$). *For an agent* $i \in S$ *we write* $x \Rightarrow_i y$ *for one of its rules.*

A belief rule 'the agent α_i believes y in context x' is an ordered pair $x \Rightarrow_i y$ with $x \in L_{AW}$ and $y \in L_W$, and a desire rule 'the agent desires y in context x' is an ordered pair $x \Rightarrow_i y$ with $x \in L_{AW}$ and $y \in L_{AW}$. It implies that the agent's beliefs are about the world ($x \Rightarrow_i p$), and not about the agent's decisions. These beliefs can be about the effects of decisions made by the agent ($a \Rightarrow_i p$) as well as beliefs about the effects of parameters set by the world ($p \Rightarrow_i q$). Moreover, the agent's desires can be about the world ($x \Rightarrow_i p$, desire-to-be), but also about the agent's decisions ($x \Rightarrow_i a$, desire-to-do). These desires can be triggered by parameters set by the world ($p \Rightarrow_i y$) as well as by decisions made by the agent ($a \Rightarrow_i y$). Modelling mental attitudes such as beliefs and desires in terms of defeasible rules results in what might be called conditional mental attitudes [4].

2.3 Agent Decisions

The belief rules are used to determine the expected consequences of a decision, where a decision δ is any subset of L_A that contains the initial decision δ^0. The set of expected consequences of this decision δ is the belief extension of $F \cup \delta$. Moreover, we consider a feasible decision as a decision that does not imply a contradiction.

Definition 3 (Decisions) *Let* $AS = \langle \{\alpha_1, \ldots, \alpha_n\}, F, B, D, \geq, \delta^0 \rangle$ *be an agent system specification. An* AS *decision profile* δ *for agents* $\alpha_1, \ldots, \alpha_n$ *is* $\delta = \langle \delta_1, \ldots \delta_n \rangle$ *where* δ_i *is a decision of agent* α_i *such that*

$$\delta_i^0 \subseteq \delta_i \subseteq L_{A_i} \quad \text{for } i = 1 \ldots n$$

A feasible decision for agent i *is* δ_i *such that*

$$E_{B_i}(F_i \cup \delta_i) \text{ is consistent}$$

A feasible decision profile is a decision profile such that

$$E_B(F \cup \delta) \text{ is consistent}$$

where we write $E_B(F \cup \delta)$ *for* $\bigcup_{i=1}^{n} E_{B_i}(F_i \cup \delta_i)$.

The following example illustrates the decisions of a single agent.

Example 2 *Let* $A_1 = \{a, b, c, d, e\}$, $W = \{p, q\}$ *and* $AS = \langle\{\alpha_1\}, F, B, D, \geq, \delta^0\rangle$ *with* $F_1 = \{\neg p\}$, $B_1 = \{c \Rightarrow q, d \Rightarrow q, e \Rightarrow \neg q\}$, $D_1 = \{\top \Rightarrow a, \top \Rightarrow b, b \Rightarrow p, \top \Rightarrow q, d \Rightarrow q\}$, $\geq_1 = \{b \Rightarrow p > \top \Rightarrow b\}$, *and* $\delta_1^0 = \{a\}$. *The initial decision* δ_1^0 *reflects that the agent has already decided in an earlier stage to reach the desire* $\top \Rightarrow a$. *Note that the consequents of all* B_1 *rules are sentences of* L_W, *whereas the antecedents of the* B_1 *rules as well as the antecedents and consequents of the* D_1 *rules are sentences of* L_{AW}. *We have due to the definition of* $E_R(S)$:
$E_B(F \cup \{a\}) = \{\neg p, a\}$
$E_B(F \cup \{a, b\}) = \{\neg p, a, b\}$
$E_B(F \cup \{a, c\}) = \{\neg p, a, c, q\}$
$E_B(F \cup \{a, d\}) = \{\neg p, a, d, q\}$
$E_B(F \cup \{a, e\}) = \{\neg p, a, e, \neg q\}$
...
$E_B(F \cup \{a, d, e\}) = \{\neg p, a, d, e, q, \neg q\}$
...
Therefore $\{a, d, e\}$ *is not a feasible AS decision profile, because its belief extension is inconsistent. Continued in Example 4.*

The following example illustrates that the set of feasible decisions of an agent may depend on the decisions of other agents.

Example 3 *Let* $A_1 = \{a\}$, $A_2 = \{b\}$, $W = \{p\}$ *and* $AS = \langle\{\alpha_1, \alpha_2\}, F, B, D, \geq, \delta^0\rangle$ *with* $F_1 = F_2 = \emptyset$, $B_1 = \{a \Rightarrow p\}$, $B_2 = \{b \Rightarrow \neg p\}$, $D_1 = \{\top \Rightarrow p\}$, $D_2 = \{\top \Rightarrow \neg p\}$, $\geq$ *is the identity relation, and* $\delta_1^0 = \delta_2^0 = \emptyset$. *We have that* $\langle\emptyset, \{b\}\rangle$ *is a feasible decision profile, but* $\langle\{a\}, \{b\}\rangle$ *is not. If* $\delta_1 = \emptyset$, *then agent* α_2 *can decide* $\delta_2 = \{b\}$. *However, if* $\delta_1 = \{a\}$, *then agent* α_2 *cannot decide so, i.e.* $\delta_2 \neq \{b\}$.

2.4 Agent preferences

In this section we introduce a way to compare decisions. We compare decisions by comparing sets of desire rules that are not reached by the decisions. Since only ordering on individual desire rules, and not ordering on sets of desire rules, are given, we first lift the ordering on individual desire rules to an ordering on sets of desire rules.

Definition 4 *Let* $AS = \langle S, F, B, D, \geq, \delta^0\rangle$ *be an agent system specification,* D_i', D_i'' *two subsets of* D_i, *and* $D_i \setminus D_i'$ *the set of* D_i *elements which are not* D_i' *elements. We have* $D_i' \succeq D_i''$ *if* $\forall d'' \in D_i'' \setminus D_i' \exists d' \in D_i' \setminus D_i''$ *such that* $d' > d''$. *We write* $D_i' \succ D_i''$ *if* $D_i' \succeq D_i''$ *and* $D_i'' \not\succeq D_i'$, *and we write* $D_i' \simeq D_i''$ *if* $D_i' \succeq D_i''$ *and* $D_i'' \succeq D_i'$.

The following propositions show that the priority relation $\succeq$ is reflexive, anti-symmetric, and transitive.

Proposition 3 *Let* $D_1 \not\subseteq D_3$ *and* $D_3 \not\subseteq D_1$. *For finite sets, the relation* $\succeq$ *is reflexive (* $\forall D\ D \succeq D$*), anti-symmetric* $\forall D_1, D_2\ (D_1 \succeq D_2\ \wedge D_2 \succeq D_1) \rightarrow D_1 = D_2$, *and transitive, i.e.* $D_1 \succeq D_2$ *and* $D_2 \succeq D_3$ *implies* $D_1 \succeq D_3$.

The desire rules are used to compare the decisions. The comparison is based on the set of unreached desires and not on the set of violated or reached desires. A desire $x \Rightarrow y$ is unreached by a decision if the expected consequences of this decision imply x but not y. The desire rule is violated or reached if these consequences imply respectively $x \wedge \neg y$ or $x \wedge y$, respectively.

Definition 5 (Comparing decisions) *Let* $AS = \langle S, F, B, D, \geq, \delta^0\rangle$ *be an agent system specification and* δ *be a AS decision. The unreached desires of decision* δ *for agent* α_i *are:*

$U_i(\delta) =$
$\{x \Rightarrow y \in D_i \mid E_B(F \cup \delta) \models x \text{ and } E_B(F \cup \delta) \not\models y\}$

Decision δ *is at least as good as decision* δ' *for agent* α_i, *written as* $\delta \geq_i^U \delta'$, *iff*

$$U_i(\delta') \succeq U_i(\delta)$$

Decision δ *dominates decision* δ' *for agent* α_i, *written as* $\delta >_i^U \delta'$, *iff*

$$\delta \geq_i^U \delta' \text{ and } \delta' \not\geq_i^U \delta$$

The following continuation of Example 2 illustrates the comparison of decisions.

Example 4 (Continued) *We have:*
$U(\{a\}) = \{\top \Rightarrow b, \top \Rightarrow q\}$,
$U(\{a, b\}) = \{b \Rightarrow p, \top \Rightarrow q\}$,
$U(\{a, c\}) = \{\top \Rightarrow q\}$,
$U(\{a, d\}) = \{\top \Rightarrow b, d \Rightarrow q\}$,
$U(\{a, e\}) = \{\top \Rightarrow b, \top \Rightarrow q\}$,
$U(\{a, b, c\}) = \{b \Rightarrow p\}$.
...
We thus have for example that the decision $\{a, c\}$ *dominates the initial decision* $\{a\}$, *i.e.* $\{a, c\} >^U \{a\}$. *There are two decisions for which their set of unreached contains only one desire. Due to the priority relation, we have that* $\{a, c\} >^U \{a, b, c\}$.

2.5 Agent games

In this subsection, we consider agents interactions based on agent system specifications, their corresponding agent decisions, and the ordering on the decisions as explained in previous subsections. Game theory is the usual tool to model the interaction between self-interested agents. Agents select optimal decisions under the assumption that other agents do likewise. This makes the definition of an optimal decision circular, and game theory therefore restricts its attention to equilibria. For example, a decision is a Nash equilibrium if no agent can reach a better (local) decision by changing its own decision. The most used concepts from game theory are Pareto efficient decisions, dominant decisions and Nash decisions. We first repeat some standard notations from game theory [1, 9].

As mentioned, we use δ_i to denote a decision of agent α_i and $\delta = \langle \delta_1, \ldots, \delta_n \rangle$ to denote a decision profile containing one decision for each agent. δ_{-i} is the decision profile of all agents except the decision of agent α_i. (δ_{-i}, δ'_i) denotes a decision profile which is the same as δ except that the decision of agent i from δ is replaced with the decision of agent i from δ'. $\delta'_i >^U_i \delta_i$ denotes that decision δ'_i is better than δ_i according to his preferences $>^U_i$ and $\delta'_i \geq^U_i \delta_i$ if better or equal. Δ is the set of all decision profiles for agents $\alpha_1, \ldots, \alpha_n$, $\Delta_f \subseteq \Delta$ is the set of feasible decision profiles, and Δ^i is the set of possible decisions for agent α_i.

Definition 6 (Game specification) *Let $AS = \langle S = \{\alpha_1, \ldots, \alpha_n\}, F, B, D, \geq, \delta^0 \rangle$ be specification of agent system in S, A_i be the set of AS feasible decisions of agent α_i according to Definition 3, $\Delta_f = A_1 \times \ldots \times A_n$, and $\geq^U_i$ be the AS preference relation of agent α_i defined on its feasible decisions according to definition 5. Then, the game specification of AS is the tuple $\langle S, \Delta_f, (\geq^U_i) \rangle$.*

We now consider different types of decision profiles which are similar to types of strategy profiles from game theory.

Definition 7 *A PS decision profile $\delta = \langle \delta_1, \ldots, \delta_n \rangle \in \Delta_f$ is:*

Pareto decision *if there is no $\delta' = \langle \delta'_1, \ldots, \delta'_n \rangle \in \Delta_f$ for which $\delta'_i >^U_i \delta_i$ for all agents α_i.*

strongly Pareto decision *if there is no $\delta' = \langle \delta'_1, \ldots, \delta'_n \rangle \in \Delta_f$ for which $\delta'_i \geq^U_i \delta_i$ for all agents α_i and $\delta'_j >^U \delta_j$ for some agents α_j.*

dominant decision *if for all $\delta' \in \Delta_f$ and for every agent i it holds: $(\delta'_{-i}, \delta_i) \geq^U_i (\delta'_{-i}, \delta'_i)$ i.e. a decision is dominant if it yields a better payoff than any other decisions regardless of what the other agents decide.*

Nash decision *if for all agents i it holds: $(\delta_{-i}, \delta_i) \geq^U_i (\delta_{-i}, \delta'_i)$ for all $\delta'_i \in \Delta^i_f$*

It is a well known fact that Pareto decisions exist (for finite games), whereas dominant decisions do not have to exist. The latter is illustrated by the following example.

Example 5 *Let α_1 and α_2 be two agents, $F_1 = F_2 = \emptyset$, and initial decisions $\delta^0_1 = \delta^0_2 = \emptyset$. They have the following beliefs en desires:*
$B_{\alpha_1} = \{a \Rightarrow p, \neg a \Rightarrow \neg p\}$
$D_{\alpha_1} = \{\top \Rightarrow p, \top \Rightarrow q\}$
$\geq_{\alpha_1} = \top \Rightarrow p > \top \Rightarrow q > \top \Rightarrow \neg q > \top \Rightarrow \neg p$
$B_{\alpha_2} = \{b \Rightarrow q, \neg b \Rightarrow \neg q\}$
$D_{\alpha_2} = \{\top \Rightarrow \neg p, \top \Rightarrow \neg q\}$
$\geq_{\alpha_2} = \top \Rightarrow \neg p > \top \Rightarrow \neg q > \top \Rightarrow q > \top \Rightarrow p$

Let Δ_f be feasible decision profiles, E_B be the outcomes of the decisions, and $U(\delta_i)$ be the set of unreached desires for agent α_i.

Δ	E_B	U_{δ_1}	U_{δ_2}
$\langle a, b \rangle$	$\{p, q\}$	$\emptyset$	$\{\top \Rightarrow \neg p, \top \Rightarrow \neg q\}$
$\langle a, \neg b \rangle$	$\{p, -q\}$	$\{\top \Rightarrow q\}$	$\{\top \Rightarrow \neg p\}$
$\langle \neg a, b \rangle$	$\{\neg p, q\}$	$\{\top \Rightarrow p\}$	$\{\top \Rightarrow \neg q\}$
$\langle \neg a, \neg b \rangle$	$\{\neg p, \neg q\}$	$\{\top \Rightarrow p, \top \Rightarrow q\}$	$\emptyset$

According to definition 5, for A_1 :
$U(\langle a, b \rangle) > U(\langle a, \neg b \rangle) > U(\langle \neg a, b \rangle) > U(\langle \neg a, \neg b \rangle)$
and for A_2 :
$U(\langle \neg a, \neg b \rangle) > U(\langle \neg a, b \rangle) > U(\langle a, \neg b \rangle) > U(\langle a, b \rangle)$.
None of these decision profiles are dominant decisions, i.e. the agents specifications has no dominant solution with respect to their unreached desires.

The following example illustrates a typical cooperation game.

Example 6 $B_1 = \{a \Rightarrow p\ ,\ b \Rightarrow \neg p \wedge q\}$, $D_1 = \{\top \Rightarrow p \wedge q\}$ $B_2 = \{c \Rightarrow q\ ,\ d \Rightarrow p \wedge \neg q\}$ $D_2 = \{\top \Rightarrow p \wedge q\}$. *The agents have a common goal $p \wedge q$, which they can only reach by cooperation.*

The following example illustrates a qualitative version of the notorious prisoner's dilemma, where the selfish behavior of individual autonomous agents leads to global bad decisions.

Example 7 *Let $A_1 = \{a\}$ (α_1 cooperates), $A_2 = \{b\}$ (α_2 cooperates), and AS be an agent system specifica-*

tion with $D_1 = \{\top \Rightarrow \neg a \wedge b, \top \Rightarrow b, \top \Rightarrow \neg(a \wedge \neg b)\}$, $D_2 = \{\top \Rightarrow a \wedge \neg b, \top \Rightarrow a, \top \Rightarrow \neg(\neg a \wedge b)\}$. *The only Nash decision is* $\{\neg a, \neg b\}$, *whereas both agents would prefer* $\{a, b\}$.

Starting from an agent system specification, we can derive the game specification and in this game specification we can use standard techniques to for example find the Pareto decisions. However, the problem with this approach is that the translation from an agent system specification to a game specification is computationally expensive. For example, a compact agent representation with only a few belief and desire rules may lead to a huge set of decisions if the number of decision variables is high.

The main challenge of qualitative game theory is therefore whether we can bypass the translation to game specification, and define properties directly on the agent system specification. For example, are there particular properties of agent system specification for which we can prove that there always exists a dominant decision for its corresponding derived game specification? A simple example is an agent system specification in which each agent has the same belief and desire rules.

In this paper we do not further pursue these issues, but we turn to our focus of interest: joint goals.

3 Joint goals

In this section we ask ourselves the question whether and how we can interpret a decision profile or equilibrium as goal-based or goal-oriented behavior. We first define decision rules and sets of decision profiles closed under indistinguishable decision profiles.

3.1 Decision rule

A decision rule maps a agent system specification to a set of possible decision profiles.

Definition 8 *A decision rule is a function from agent system specifications to sets of feasible decision profiles.*

Decision theory prescribes a decision maker to select the optimal or best decision, which can be defined as a decision that is not dominated. Is there an analogous prescription for societies of agents? A set of *cooperating* agents has to select an optimal or Pareto decision. We call such cooperating agents a BD rational society of cooperating agents.

Definition 9 *A BD rational society of cooperating agents is a set of agents, defined by an agent system specification AS, that selects a Pareto AS decision.*

In this paper we consider decision rules based on unreached desires. We therefore assume that a decision rule cannot distinguish between decision profiles δ_1 and δ_2 such that $\delta_1 \sim^U \delta_2$. We say that two decision profiles δ and δ' are indistinguishable if $U(\delta) = U(\delta')$, and we call a set of decisions U-closed if the set is closed under indistinguishable decision profiles.

Definition 10 *Let* $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ *be an agent system specification and* Δ *a set of AS decision profiles.* Δ *is U-closed if* $\delta \in \Delta$ *implies* $\delta' \in \Delta$ *for all AS decision profiles* δ' *such that* $U(\delta) = U(\delta')$.

An example of a decision rule is the function that maps agent system specifications to Pareto decision profiles (a BD rational decision rule). Another example is a function that maps agent system specifications to Nash equilibria if they exist, otherwise to Pareto decisions.

3.2 Goals

In this section we show that every society of agents can be understood as planning for joint goals, whether the decision is reached by cooperation or is given by a (e.g. Nash) equilibrium. We define not only goals which must be reached, called positive goals, but we add negative goals. Negative goals are defined in the following definition as states the agent has to avoid. They function as constraints on the search process of goal-based decisions.

Definition 11 (Goal-based decision) *Let* $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ *be an agent system specification, and the so-called positive joint goal set* G^+ *and the negative joint goal set* G^- *be subsets of* L_{AW}. *A decision* δ *is a* $\langle G^+, G^- \rangle$ *decision if* $E_B(F \cup \delta) \models_{AW} G^+$ *and for each* $g \in G^-$ *we have* $E_B(F \cup \delta) \not\models_{AW} g$.

Joint goals are defined with respect to a set of decision profiles. The definition of Δ joint goal set encodes a decision profile together with its indistinguishable decision profiles as a positive and negative goal set.

Definition 12 (Δ goal set) *Let* $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ *be an agent system specification and* Δ *an U-closed set of feasible decisions. The two sets of formulas* $\langle G^+, G^- \rangle \subseteq (L_{AW}, L_{AW})$ *is a* Δ *joint goal set of AS if there is an AS decision* $\delta \in \Delta$ *such that*

$$G^+ = \{y \mid x \Rightarrow y \in D, E_B(F \cup \delta) \models_{AW} x \wedge y\}$$

$$G^- = \{x \mid x \Rightarrow y \in D, E_B(F \cup d) \not\models_{AW} x\}$$

$\langle G^+, G^- \rangle \subseteq (L_{AW}, L_{AW})$ is a feasible joint goal set of AS if there is an U-closed set of feasible decisions Δ such that $\langle G^+, G^- \rangle \subseteq (L_{AW}, L_{AW})$ is a Δ joint goal set of AS. $\langle G^+, G^- \rangle \subseteq (L_{AW}, L_{AW})$ is a joint goal set of AS if there is a set $D' \subseteq D$ such that

$$G^+ = \{y \mid x \Rightarrow y \in D'\}$$

$$G^- = \{x \mid x \Rightarrow y \in D'\}$$

The first part of the representation theorem follows directly from the definitions.

Proposition 4 *Let $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ be an agent system specification and Δ be an U-closed set of feasible decision profiles. For a decision profile $\delta \in \Delta$ of AS there is a Δ joint goal set $\langle G^+, G^- \rangle$ of AS such that δ is a $\langle G^+, G^- \rangle$ decision.*

Proof. Follows directly from the definitions.

Proposition 5 *Let $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ be an agent system specification and Δ an U-closed set of feasible decision profiles. For a Δ joint goal set $\langle G^+, G^- \rangle$ of AS, a $\langle G^+, G^- \rangle$ decision is a Δ decision.*

Proof. Follows from U-closed property.

The representation theorem is a combination of Proposition 4 and 5.

Theorem 1 *Let $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ be an agent system specification and Δ an U-closed set of feasible decision profiles. A decision profile δ is in Δ if and only if there is a Δ goal set $\langle G^+, G^- \rangle$ of AS such that δ is a $\langle G^+, G^- \rangle$ decision profile.*

The second theorem follows from the first one.

Theorem 2 *Let $AS = \langle S, F, B, D, \geq, \delta^0 \rangle$ be an agent system specification. A decision profile δ is a feasible AS decision profile if and only if there is a feasible goal set $\langle G^+, G^- \rangle$ of AS such that δ is a $\langle G^+, G^- \rangle$ decision profile.*

Consider a society of agent that tries to determine its Pareto decision profiles. The game specifications suggest the following algorithm:

```
calculate all decision profiles
for all decision profiles,
  calculate consequences
order all decision profiles
```

The goal-based representation suggest an alternative approach:

```
calculate all joint goals
filter feasible joint goals
for each feasible joint goal set
  find goal-based decision profiles
order these decision profiles
```

In other words, the goal-based representation suggests to calculate the joint goals first. However, the problem to calculate these joint goals is still computationally hard. There are two ways to proceed:

- Define heuristics for the optimization problem;
- Find a fragment of the logic, such that the optimization becomes easier.

For example, consider the following procedure to find (positive) goals:

$$G \subseteq E_{B \cup D}(F \cup \delta^0)$$

This procedure is not complete, because it does not take effects of actions into account. Thus, in the general case it can be used as a heuristic. Moreover, it is complete for the fragment in which the belief rules do not contain effects of actions, i.e. $B \subseteq S \times L_W \times L_W$.[1]

4 Concluding remarks

In this paper we have defined a qualitative decision and game theory in the spirit of classical decision and game theory. The theory illustrates the micro-macro dichotomy by distinguishing the optimization problem from game theoretic equilibria. We also showed that any group decision, whether based on optimization or on an equilibrium, can be represented by positive and negative goals.

We think that the method of this paper is more interesting than its formal results. The decision and game theory are based on several ad hoc choices which need further investigation. For example, the desire rules are defeasible but the belief rules are not (the obvious extension leads to wishful thinking problems as studied in [14, 3]). However, the results suggest that any group decision can be understood as reaching for goals. We hope that further investigations along this line brings the theories and tools used for individual agents and multi agent systems closer together.

[1] We cannot add beliefs on decision variables, as will be clear in the next section. Suppose $A = \{a\}$ and $B = \{\top \rightarrow a\}$. Clearly all decisions should be considered. However, all decision profiles given in Definition 3 imply a and thus all decisions not implying a would be excluded.

References

[1] K. Binmore. *Fun and Games : A Text on Game Theory.* 1992.

[2] C. Boutilier. Toward a logic for qualitative decision theory. In *Proceedings of the KR'94*, pages 75–86, 1994.

[3] J. Broersen, M. Dastani, , and L. van der Torre. Resolving conflicts between beliefs, obligations, intentions and desires. In *Symbolic and Quantitative Approaches to Reasoning and Uncertainty. Proceedings of ECSQARU'01*, LNCS 2143, pages 568–579. Springer, 2001.

[4] J. Broersen, M. Dastani, Z. Huang, J. Hulstijn, and L. van der Torre. The BOID architecture: Conflicts between beliefs, obligations, intentions and desires. In *Proceedings of he Fifth International Conference on Autonomous Agents (AA2001)*, pages 9–16. ACM Press, 2001.

[5] P.R. Cohen and H.J. Levesque. Intention is choice with commitment. *Artificial Intelligence*, 42:213–261, 1990.

[6] J. Doyle and R. Thomason. Background to qualitative decision theory. *AI magazine*, 20(2), summer 1999.

[7] J. Lang. Conditional desires and utilities - an alternative approach to qualitative decision theory. In *In Proceedings of the European Conference on Artificial Intelligence (ECAI'96)*, pages 318–322, 1996.

[8] A. Newell. The knowledge level. *Artificial Intelligence*, 1982.

[9] Martin J. Osborne and Ariel Rubenstein. *A Course in Game Theory.* The MIT Press, Cambridge, Massachusetts, 1994.

[10] A. Rao and M. Georgeff. Modeling rational agents within a BDI architecture. In *Proceedings of the KR91*, 1991.

[11] L. Savage. *The foundations of statistics.* 1954.

[12] H. A. Simon. *The Sciences of the Artificial.* MIT Press, Cambridge, MA, second edition, 1981.

[13] R. Thomason. Practical reason. Presentation at PRR'99 workshop at IJCAI'99.

[14] R. Thomason. Desires and defaults: A framework for planning with inferred goals. In *Proceedings KR 2000*, pages 702–713, 2000.

[15] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior.* Princeton University Press, Princeton, NJ, 1 edition, 1944.