Acyclic Argumentation: Attack = Conflict + Preference

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Abstract. In this paper we study the fragment of Dung's argumentation theory in which the strict attack relation is acyclic. We show that every attack relation satisfying a particular property can be represented by a symmetric conflict relation and a transitive preference relation in the following way. We define an instance of Dung's abstract argumentation theory, in which 'argument A attacks argument B' is defined as 'argument A conflicts with argument B' and 'argument A is at least as preferred as argument B', where the conflict relation is symmetric and the preference relation is transitive. We show that this new preference-based argumentation theory characterizes the acyclic strict attack relation, in the sense that every attack relation defined as such a combination satisfies the property, and for every attack relation satisfying the property we can find a symmetric conflict relation and a transitive preference relation satisfying the equation.

1 Acyclic argumentation framework

Argumentation is a reasoning model based on constructing arguments, determining potential conflicts between arguments and determining acceptable arguments. Dung's framework [7] is based on a binary attack relation among arguments. We restrict ourselves to *finite* argumentation frameworks, i.e., when the set of arguments $\mathcal{A}$ is *finite*.

Definition 1 (Argumentation framework) *An argumentation framework is a tuple $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments and $\mathcal{R}$ is a binary attack relation defined on $\mathcal{A} \times \mathcal{A}$.*

An acyclic argumentation framework is an argumentation framework in which the attack relation is acyclic, a symmetric argumentation framework is an argumentation framework in which the attack relation is symmetric, *etc*. In this paper we define an acyclic strict attack relation as follows. Assume the attack relation is such that there is an attack path where argument A_1 attacks argument A_2, argument A_2 attacks argument A_3, etc, and argument A_n attacks argument A_1, then we have that all the arguments in the attack path attack the previous one. Consequently, if argument A strictly attacks B if A attacks B and not vice versa, then the strict attack relation is acyclic.

Definition 2 (Acyclic argumentation framework) *A strictly acyclic argumentation framework is an argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ in which the attack relation $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ satisfies the following property:*

> *If there is a set of attacks $A_1 \mathcal{R} A_2$, $A_2 \mathcal{R} A_3$, $\cdots$, $A_n \mathcal{R} A_1$ then we have that $A_2 \mathcal{R} A_1$, $A_3 \mathcal{R} A_2$, $\cdots$, $A_1 \mathcal{R} A_n$.*

The semantics of Dung's argumentation framework are based on the two notions of defence and conflict free.

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Definition 3 (Defence) *A set of arguments $\mathcal{S}$ defends A iff for each argument B of $\mathcal{A}$ which attacks A, there is an argument C in $\mathcal{S}$ which attacks B.*

Definition 4 (Conflict-free) *Let $\mathcal{S} \subseteq \mathcal{A}$. The set $\mathcal{S}$ is conflict-free iff there are no $A, B \in \mathcal{S}$ such that $A \mathcal{R} B$.*

The following definition summarizes various semantics of acceptable arguments proposed in the literature. The output of the argumentation framework is derived from the set of selected acceptable arguments with respect to an acceptability semantics.

Definition 5 (Acceptability semantics) *Let $\mathcal{S} \subseteq \mathcal{A}$.*

- *$\mathcal{S}$ is* admissible *iff it is conflict-free and defends all its elements.*
- *A conflict-free $\mathcal{S}$ is a* complete extension *iff we have $\mathcal{S} = \{A \mid \mathcal{S}$ defends $A\}$.*
- *$\mathcal{S}$ is a* grounded extension *iff it is the smallest (for set inclusion) complete extension.*
- *$\mathcal{S}$ is a* preferred extension *iff it is a maximal (for set inclusion) complete extension.*
- *$\mathcal{S}$ is a* stable extension *iff it is a preferred extension that attacks all arguments in $\mathcal{A} \backslash \mathcal{S}$.*

For the general case, in which the attack relation can be any relation, many properties and relations among these semantics have been studied. However, of instances of Dung's argumentation framework, not much is known (with the exception of symmetric argumentation frameworks, see [6]). In this paper we consider acyclic argumentation.

2 Conflict+preference argumentation framework

Consider an argumentation theory in which each argument is represented by a propositional formula, and 'argument A attacks argument B' is defined as '$A \wedge B$ is inconsistent'. Unfortunately, such a simple argumentation theory is not very useful, since the attack relation is symmetric, and the various semantics reduce to, roughly, one of the following two statements: 'an argument is acceptable iff it is part of all/some maximal consistent subsets of the set of arguments.' However, such a simple argumentation theory is useful again when we add the additional condition that argument A is at least as preferred as argument B. We start with some definitions concerning preferences.

Definition 6 *A pre-order on a set $\mathcal{A}$, denoted $\succeq$, is a reflexive and transitive relation. $\succeq$ is total if it is complete and it is partial if it is not. The notation $A_1 \succeq A_2$ stands for A_1 is at least as preferred as A_2. $\succ$ denotes the order associated with $\succeq$. $A_1 \succ A_2$ means that we have $A_1 \succeq A_2$ without $A_2 \succeq A_1$.*

The new preference-based argumentation framework considers a conflict and a preference relation. The conflict relation should not be interpreted as an attack relation, since a conflict relation is symmetric, and an attack relation is usually asymmetric.

Definition 7 (Conflict+preference argumentation framework) *A conflict+preference argumentation framework is a triplet $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{C}$ is a binary symmetric conflict relation defined on $\mathcal{A} \times \mathcal{A}$ and $\succeq$ is a (total or partial) pre-order (preference relation) defined on $\mathcal{A} \times \mathcal{A}$.*

Starting with a set of arguments, a symmetric conflict relation, and a preference relation, we combine this conflict relation with preference relation to compute Dung's attack relation. Then we use any of Dung's semantics to define the acceptable set of arguments. In contrast to most other approaches [1, 8] (but see [2, 3] for exceptions), our approach to reason about preferences in argumentation does not refer to the internal structure of the arguments.

Definition 8 *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ a conflict+preference argumentation framework. We say that $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents $\langle \mathcal{A}, \mathcal{R} \rangle$ iff for all arguments A and B of $\mathcal{A}$, we have $A\ \mathcal{R}\ B$ iff $A\ \mathcal{C}\ B$ and $A \succeq B$. We also say that $\mathcal{R}$ is represented by $\mathcal{C}$ and $\succeq$.*

Since the attack relation is defined on the basis of conflict $\mathcal{C}$ and preference relation $\succeq$, also the other relations defined by Dung are reused by the conflict+preference argumentation framework. For example, to compute the grounded semantics of the conflict+preference framework, first compute the attack relation, and then compute the grounded semantics as in the general case.

3 Acyclic attack = conflict + preference

To prove that acyclic attacks are characterized by conflicts and preferences, we have to show that the implication holds in both ways. We first show that the implication from right to left holds, which is the easiest direction.

Lemma 1 *If the conflict+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents the argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$, then $\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework (in the sense of Definition 2).*

Proof. *Assume a set of attacks $A_1 \mathcal{R} A_2$, $A_2 \mathcal{R} A_3$, ..., $A_{n-1} \mathcal{R} A_n$, $A_n \mathcal{R} A_1$. Since $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents $\langle \mathcal{A}, \mathcal{R} \rangle$, we have also $A_i\ \mathcal{C}\ A_{(i \bmod n)+1}$ and $A_i \succeq A_{(i \bmod n)+1}$. Due to symmetry of $\mathcal{C}$, we also have $A_{(i \bmod n)+1}\ \mathcal{C}\ A_i$. Moreover, due to transitivity of $\succeq$, we have $A_{(i \bmod n)+1} \succeq A_i$ for $1 \leq i \leq n$. Consequently we have $A_{(i \bmod n)+1} \mathcal{R} A_i$, and thus $\langle \mathcal{A}, \mathcal{R} \rangle$ is strictly acyclic.*

Now we show that the implication from left to right holds. We prove this lemma by construction: given an acyclic argumentation framework, we construct a conflict+preference framework representing it.

Lemma 2 *If $\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework, then there is a conflict+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ that represents it.*

Proof. *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be a strictly acyclic argumentation framework. Moreover, consider a conflict+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ defined as follows:*

- *$\mathcal{C} = \{(a, b) \mid (a, b) \in \mathcal{R}$ or $(b, a) \in \mathcal{R}\}$ is the symmetric closure of $\mathcal{R}$*
- *$\succeq$ is the transitive and reflexive closure of $\mathcal{R}$*

$\mathcal{R} \subseteq \mathcal{C} \cap \succeq$ by construction. The other direction we prove by contradiction, so assume $A \mathcal{C} B$ and $A \succeq B$ without $A \mathcal{R} B$. Since $A \mathcal{C} B$ we have also $B \mathcal{C} A$ (since $\mathcal{C}$ is symmetric) and we must have $B \mathcal{R} A$ (due to $A \mathcal{C} B$ and $B \mathcal{C} A$ we have either $A \mathcal{R} B$ or $B \mathcal{R} A$). Since $A \succeq B$ we must have either $A = B$ (added by reflexive closure) or there is an attack path from A to B (added by transitivity). In both cases we obtain a contradiction:

1. *If $A = B$ then $B \mathcal{R} A$ without $A \mathcal{R} B$ is directly a contradiction.*
2. *If there is a path from A to B, then together with $B \mathcal{R} A$ there is a cycle, and due to the acyclicity property we have $A \mathcal{R} B$, which contradicts the assumption.*

Summarizing, strictly acyclic argumentation frameworks are characterized by conflict+preference argumentation frameworks.

Theorem 1 *$\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework if and only if there is a conflict+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ that represents it.*

4 Concluding remarks

The preference-based argumentation theory introduced in this paper is a variant of the preference based framework of Amgoud and Cayrol [1], who define that argument A attacks argument B if A defeats B and B is not preferred to A. Note that our representation theorem does not hold for such a definition. As far as we know it is an open problem which properties the attack relation satisfies (if the defeat relation is symmetric).

Another subject for further research is how to use the strictly acyclic or conflict+preference frameworks. It may also be worthwhile to consider the generalizations of Dung's framework in propositional argumentation [4, 5].

Finally, a subject for further study is the complexity of the argumentation frameworks discussed in this paper. Is the computation of acceptable arguments in some sense easier or more efficient for strictly acyclic argumentation frameworks? Also, can we find more efficient or anytime algorithms for these frameworks?

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