On the Acceptability of Incompatible Arguments

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Abstract. In this paper we study the acceptability of incompatible arguments within Dung's abstract argumentation framework. As an example we introduce an instance of Dung's framework where arguments are represented by propositional formulas and an argument attacks another one when the conjunction of their representations is inconsistent, which we characterize as a kind of symmetric attack. Since symmetric attack is known to have the drawback to collapse the various argumentation semantics, we consider also two variations. First, we consider propositional arguments distinguishing support and conclusion. Second, we introduce a preference ordering over the arguments and we define the attack relation in terms of a symmetric incompatibility relation and the preference relation. We show how to characterize preference-based argumentation using a kind of acyclic attack relation.

1 Introduction

Dung's abstract argumentation framework [10] is based only on sets (whose elements are called *arguments*) carrying a binary relation (called the *attack* relation). Due to this abstract perspective, it can and has been used in several ways, for example as a general framework for non-monotonic reasoning, for argumentation, and as a component in agent communication, dialogue, or decision making.

However, the increasing popularity of argumentation-based systems has revealed a dilemma. On the one hand many users appreciate the abstract arguments of Dung's framework, enabling them to reason about arguments without being forced to use a prescribed representation such as rules. On the other hand, in general they find it difficult to think in terms of attack relations. For example, does the argument

> "The soccer game is going to be very interesting, many people will watch the game because it is Barcelona against Arsenal"

attack the argument

> "The soccer game is going to be boring, because the first half was already won by 6-0"

or vice versa? Typically, users specify only whether two arguments are incompatible, i.e., whether two arguments cannot be held together. Incompatibility means here simply that a person cannot forward the two arguments without contradicting himself. This means that the attack relation is symmetric, because if argument A is incompatible with argument B, then argument B is incompatible with argument A.

In this paper we are therefore interested in incompatibility of arguments. However, Besnard and Hunter [7] and Coste-Marquis *et al.* [9] have shown that symmetric attack relations – as represented by propositional argumentation – are not very expressive in the sense that the various semantics of Dung's argumentation framework collapse to selecting either a maximal conflict-free subset of arguments, or the intersection of the maximal conflict-free subsets. We therefore propose that argumentation based systems should use the following argumentation specification:

> **Argumentation specification for argumentation based systems.** The user specifies abstract arguments, a symmetric incompatibility relation on the arguments, and a preference relation over the arguments. The system calculates first the attack relation from the incompatibility relation and the preference relation, and thereafter the acceptable arguments using one of Dung's semantics.

This proposed specification leaves us the freedom to define the properties of the preference relation and the way to define the attack relation from the other two relations. For the preference relation most users prefer to use a transitive relation, so, for example, a partial order or a total order. For the combination we can define, for example, that argument A attacks argument B when A and B are incompatible and A is at least as preferred as B [11], or B is not preferred to A [2, 5].

The choice among these alternatives may depend on which alternative is most intuitive for the user, but it seems that there is no strong preference for either of them. It may depend also on the expressive power of the alternatives: if one representation can represent a larger range of attack relations, it may be a better basis for argumentation based systems. For example, do we again have a collapse of the various semantics, as in the case of symmetric attack relations without a preference relation, or can we represent any kind of attack relation and do we thus cover the full range of possibilities? As we show later in this paper, the answer is often somewhere in the middle: we do not have a collapse of the various semantics, but we can represent only a subset of all possible attack relations.

In this paper we are therefore interested in characterizing various kinds of alternative argumentation frameworks, in the following sense. Given an argumentation framework consisting of a set of arguments and one or more other mathematical elements, we define a mapping from the argumentation framework to a Dung's abstract argumentation framework. Then we say that the alternative framework is characterized by a property if the following two conditions hold. First, for every possible mapping, the property holds for Dung's framework. Second, for every instance of Dung's framework satisfying the property, there is an instance of the alternative framework that is mapped onto it. We say also that Dung's theory together with the property is represented by the alternative framework.

For example, we consider what we call a propositional argumentation framework in which each argument A is represented by a formula $prop(A)$ from propositional logic, and we define the mapping to Dung's framework as follows: argument A attacks argument B if $prop(A) \wedge prop(B)$ is inconsistent. We then characterize the propositional argumentation theory by symmetric attack, if argument A attacks argument B, then argument B attacks argument A, together with the property that if argument A attacks itself, then it attacks all other arguments too. Thus, this particular kind of symmetric attack can be represented by propositional argumentation.

In particular, in this paper we address the following two questions:

1. Can the collapse of the argumentation semantics be avoided when we represent arguments by pairs $\langle H, h\rangle$, where H is a set of propositional formulas supporting the formula h (i.e., H logically implies h in propositional logic), a kind of propositional argumentation promoted, for example, by Amgoud and Cayrol [2]?
2. How do we characterize an argumentation framework with an incompatibility relation together with a preference relation over arguments, and the mapping that argument A attacks argument B if and only if A and B are incompatible, and B is not preferred to A, as suggested by Amgoud and Cayrol [2] and Bench-Capon [5]?

The results in this paper show that the attack relations which cannot be represented contain a particular kind of cycles of attacking arguments. Attack cycles among arguments have raised already considerable attention in the argumentation literature, in particular due to the fact that odd cycles have a distinct behavior from even cycles, in the sense that the former arguments are not contained in any acceptable preferred or stable semantics, whereas some of the latter are. However, the problem with this distinct behavior is that for users it is very hard to distinguish, for example, a 5-cycle from a 6-cycle. The fact that such cycles can no longer be represented, may be an alternative explanation why it is easier for users to represent an incompatibility relation together with a preference relation, than an attack relation.

The layout of this paper is as follows. In Section 2 we introduce Dung's framework, propositional and extended propositional argumentation, and we characterize them. In Section 3 we introduce preference-based argumentation and we characterize it.

2 Propositional argumentation

In this section we repeat Dung's argumentation framework, we discuss basic propositional argumentation and we characterize it as a kind of symmetric attack. Then we discuss an extended form of propositional argumentation and we characterize it. Finally we introduce and discuss a notion of closure under supported arguments, and we show that under this assumption extended propositional argumentation collapses to propositional argumentation.

2.1 Dung's framework

Argumentation is a reasoning model which consists of constructing arguments, determining potential conflicts between arguments, and selecting acceptable arguments. Dung's framework [10] is based on a binary attack relation among arguments.

Definition 1 (Argumentation framework) *An argumentation framework is a tuple $\langle \mathcal{A}, \mathcal{R}\rangle$ where $\mathcal{A}$ is a set of arguments and $\mathcal{R}$ is a binary attack relation on $\mathcal{A} \times \mathcal{A}$.*

The semantics of Dung's argumentation framework is based on the two notions of defence and conflict-freeness.

Definition 2 (Defence) *A set of arguments $\mathcal{S}$ defends an argument A iff for each argument B of $\mathcal{A}$ which attacks A, there is an argument C in $\mathcal{S}$ which attacks B.*

Definition 3 (Conflict-free) *A set of arguments $\mathcal{S}$ is conflict-free iff there are no $A, B \in \mathcal{S}$ such that $A\mathcal{R}B$.*

The following definition summarizes various semantics of acceptable arguments proposed in the literature. The output of the argumentation framework is derived from the set of acceptable arguments which are selected with respect to an acceptability semantics.

Definition 4 (Acceptability semantics) *Let $\mathcal{S} \subseteq \mathcal{A}$.*

- *$\mathcal{S}$ is* admissible *iff it is conflict-free and defends all its elements.*
- *A conflict-free $\mathcal{S}$ is a* complete extension *iff $\mathcal{S} = \{A \mid \mathcal{S}$ defends $A\}$.*
- *$\mathcal{S}$ is a* grounded extension *iff it is the smallest (for set inclusion) complete extension.*
- *$\mathcal{S}$ is a* preferred extension *iff it is a maximal (for set inclusion) complete extension.*
- *$\mathcal{S}$ is a* stable extension *iff it is a preferred extension that attacks all arguments in $\mathcal{A}\backslash\mathcal{S}$.*

Many properties and relations among these semantics have been studied by Dung and others.

2.2 Basic propositional argumentation

In basic propositional argumentation, each argument is represented by a propositional formula.

Definition 5 *Let L be a language of propositional logic. A basic propositional argumentation framework is a tuple $\langle \mathcal{A}, prop \rangle$ where $\mathcal{A}$ is a set of arguments and $prop$ is a function from $\mathcal{A}$ to L.*

An argument represented by a propositional formula p attacks an argument represented by q if and only if $p \wedge q$ is inconsistent.

Definition 6 *$\langle \mathcal{A}, prop \rangle$ represents $\langle \mathcal{A}, \mathcal{R} \rangle$ if and only if for all $A, B \in \mathcal{A}$, we have $A \mathcal{R} B$ iff $prop(A) \wedge prop(B)$ is inconsistent in propositional logic. We say also that $\mathcal{R}$ is represented by $prop$.*

Example 1. Consider a propositional argumentation framework $\langle \{A, B, C, D\}, prop \rangle$ with $prop(A) = p, prop(B) = \neg p \wedge q$, $prop(C) = q$ and $prop(D) = r \wedge \neg r$. We have that A attacks B and vice versa, D attacks all other arguments and vice versa, and no other attack relations hold.

Roughly, basic propositional argumentation corresponds to symmetric argumentation.

Theorem 1 *If $\mathcal{A}$ is finite, then $\langle \mathcal{A}, \mathcal{R} \rangle$ can be represented by a basic propositional argumentation framework if and only if the following two properties hold:*

1. *$\mathcal{R}$ is symmetric.*
2. *For all arguments A, B, if $A\mathcal{R}A$, then $A\mathcal{R}B$.*

Proof. *Soundness. Let $\langle \mathcal{A}, prop \rangle$ be a basic propositional argumentation theory representing $\langle \mathcal{A}, \mathcal{R} \rangle$. Symmetry holds, because $prop(A) \wedge prop(B)$ is inconsistent if and only if $prop(B) \wedge prop(A)$ is inconsistent (with respect to propositional logic). Moreover, an argument A attacks itself, $A\mathcal{R}A$, if and only if $prop(A)$ is inconsistent. However, if $prop(A)$ is inconsistent, then $prop(A) \wedge prop(B)$ is inconsistent as well, thus $A\mathcal{R}B$.*

Completeness. We prove it by construction. Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an arbitrary argumentation theory satisfying that $\mathcal{R}$ is symmetric and $\forall A, B \in \mathcal{A}$, if $A\mathcal{R}A$ then $A\mathcal{R}B$. Let $\langle A_1, \ldots, A_{|\mathcal{A}|} \rangle$ be a sequence of all the elements of $\mathcal{A}$. Moreover, consider $|\mathcal{A}|$ propositional atoms p_i, define

$$P = \bigwedge_{1 \leq i,j \leq |\mathcal{A}|} \{\neg(p_i \wedge p_j) \mid A_i \mathcal{R} A_j\},$$

$$prop(A_i) = p_i \wedge P.$$

We prove that this basic argumentation framework represents $\langle \mathcal{A}, \mathcal{R} \rangle$ by showing that $\forall A, B \in \mathcal{A}$, $A\mathcal{R}B$ if and only if $prop(A) \wedge prop(B)$ is inconsistent.

Let p and q be the propositional atoms associated to A and B respectively. Suppose that $A\mathcal{R}B$. Then $prop(A) = p \wedge P$ and $prop(B) = q \wedge P$. Since $A\mathcal{R}B$ then $\neg(p \wedge q)$ belongs to P. Thus $prop(A) \wedge prop(B)$ is inconsistent.

Suppose that $prop(A) \wedge prop(B)$ is inconsistent. This means that $(p \wedge P) \wedge (q \wedge P)$ is inconsistent. Since p and q are propositional atoms and thus positive formulas, this can hold only if either:

1. *$\neg(p \wedge q)$ belongs to P, which means that $A\mathcal{R}B$. By symmetry of $prop(A) \wedge prop(B)$ we have also $B\mathcal{R}A$.*
2. *We have $p_i = p_j = p$ or $p_i = p_j = q$ for some p_i and p_j, which means either A or B attacks itself. But then by the second rule we have either $A\mathcal{R}B$ or $B\mathcal{R}A$, and by symmetry we have in both cases $A\mathcal{R}B$ and $B\mathcal{R}A$.*

Consequently, in both cases we have $A\mathcal{R}B$. This concludes the proof.

2.3 Symmetric attack relations

Besnard and Hunter [7] and Coste-Marquis *et al.* [9] show that only two distinct forms of acceptability are possible when the considered frameworks are symmetric. Those forms of acceptability are quite rudimentary, but tractable; this contrasts with the general case where all the forms of acceptability are intractable (except the ones based on grounded extensions).

Theorem 2 *[9] Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework where $\mathcal{R}$ is symmetric, non-empty, and irreflexive. $A \in \mathcal{A}$ belongs to every preferred (equivalently, stable) extension of $\langle \mathcal{A}, \mathcal{R} \rangle$ if and only if there is no $B \in \mathcal{A}$ such that $B\mathcal{R}A$.*

Coste-Marquis *et al.* summarize that there are at most two distinct forms of acceptability for symmetric argumentation frameworks: all the forms of skeptical acceptability coincide with the notion of acceptability with respect to the grounded extension, and credulous acceptability with respect to preferred extensions coincides with credulous acceptability with respect to stable extensions.

Consequently, one has to consider more general acceptability notions if one wants to get more than one semantics, as we do here; indeed, skeptical acceptability is rather poor since it characterizes as acceptable only those arguments which are not attacked. For these reasons, Coste-Marquis *et al.* turn to acceptability concepts for *sets* of arguments, that is, the question is to determine whether or not it is reasonable to accept some arguments together.

2.4 Extended propositional argumentation

Arguments can be represented by pairs $\langle H, h\rangle$, where $H \cup \{h\}$ is a set of propositional formulas [12]. We call H the support and h the conclusion of the argument.

Definition 7 *Let L be a language of propositional logic. An extended propositional argumentation framework is a tuple $\langle \mathcal{A}, sup, con\rangle$ where $\mathcal{A}$ is a set (of arguments), $sup : \mathcal{A} \rightarrow 2^L$ is a (support) function from arguments to sets of formulas, and $con : \mathcal{A} \rightarrow L$ is a (conclusion) function from arguments to formulas such that for all $A \in \mathcal{A}$, $sup(A)$ (proposition) logically implies $con(A)$.*

We have two kinds of attack. An argument $\langle H, h\rangle$ rebuts $\langle K, k\rangle$ if and only if $h \wedge k$ is inconsistent, and the former undercuts the latter if and only if $\{h\} \cup K$ is inconsistent. Note that using these definitions of rebut and undercut, due to the fact that $sup(A)$ logically implies $con(A)$, we have that if A rebuts B, then also A undercuts B (but not necessarily vice versa). We therefore consider undercutting only in the following definition of representation.

Definition 8 *$\langle \mathcal{A}, sup, con\rangle$ represents $\langle \mathcal{A}, \mathcal{R}\rangle$ if and only if for all $A, B \in \mathcal{A}$, we have $A \mathcal{R} B$ iff $\{con(A)\} \cup sup(B)$ is inconsistent in propositional logic.*

Example 2. Let $\langle \{A, B, C\}, sup, con\rangle$ be an extended propositional argumentation theory with $sup(A) = \{\neg p \wedge q\}$, $con(A) = q$, $sup(B) = \{p \wedge \neg q\}$ and $con(B) = p$, $sup(C) = \{p \vee r, \neg r \vee s, \neg s\}$ and $con(C) = p$. We may represent this theory also by $A = \langle \{\neg p \wedge q\}, q\rangle$, $B = \langle \{p \wedge \neg q\}, p\rangle$, and $C = \langle \{p \vee r, \neg r \vee s, \neg s\}, p\rangle$. Note that $\langle \{\neg p \wedge q\}, q\rangle$ is an argument whereas $\langle \{\neg p, q\}, q\rangle$ is not due to the minimality criterion of the support of an argument. We have argument A attacks argument B and vice versa, argument C attacks argument A, and no other attack relations hold.

Theorem 3 *If $\mathcal{A}$ is finite, then $\langle \mathcal{A}, \mathcal{R}\rangle$ can be represented by an extended propositional argumentation framework if and only if for all arguments $A, B \in \mathcal{A}$, if $A\mathcal{R}A$, then $B\mathcal{R}A$.*

Proof. Soundness. Let $\langle \mathcal{A}, sup, con\rangle$ be an extended propositional argumentation framework representing $\langle \mathcal{A}, \mathcal{R}\rangle$. Suppose that $A\mathcal{R}A$. Then $\{con(A)\} \cup sup(A)$ is inconsistent. Following Definition 7, $sup(A)$ logically implies $con(A)$. Hence, $sup(A)$ must be *inconsistent ($con(A)$ is not necessarily inconsistent). But then we have for all $B \in \mathcal{A}$ that $\{con(B)\} \cup sup(A)$ is inconsistent, i.e., that $B\mathcal{R}A$.*

Completeness. We prove it by construction. Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an arbitrary argumentation framework satisfying the condition that if $A\mathcal{R}A$, then $B\mathcal{R}A$. Let $\langle A_1, \ldots, A_{|\mathcal{A}|} \rangle$ be a sequence of all the elements of $\mathcal{A}$. Moreover, consider $|\mathcal{A}|$ propositional positive atoms p_i, define

$$P_i = \{\neg(p_i \wedge p_j) \mid 1 \leq j \leq |\mathcal{A}|, A_j \mathcal{R} A_i\},$$

$$sup(A_i) = \{p_i\} \cup P_i,$$

$$con(A_i) = p_i.$$

We prove that the above extended propositional framework represents the argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$, i.e. $\forall A, B \in \mathcal{A}$, $A\mathcal{R}B$ iff $\{con(A)\} \cup sup(B)$ is inconsistent.

Suppose that $A\mathcal{R}B$. Let p and q be the propositional atoms associated to A and B respectively. We have conclusion $con(A) = p$ and support $sup(B) = \{q\} \cup P$ with $P = \{\neg(q \wedge p_j) \mid A_j \mathcal{R} B\}$. We have that $\neg(q \wedge p) \in P$ due to $A\mathcal{R}B$. Then it follows that $\{con(A)\} \cup sup(B)$ is inconsistent.

Suppose now that $\{con(A)\} \cup sup(B)$ is inconsistent. This means that we have that $\{p, q\} \cup \{\neg(q \wedge p_j) \mid A_j \mathcal{R} B\} = \{p, q \neg(q \wedge p_1), \neg(q \wedge p_2), \cdots, \neg(q \wedge p_m)\}$ is inconsistent. Since p, q and p_j are propositional atoms, and thus positive formulas, this formula is inconsistent if and only if either:

1. *there exists p_j ($j = 1, \cdots, m$) such that $p_j = p$. Then $A\mathcal{R}B$, or*
2. *there exists p_j ($j = 1, \cdots, m$) such that $p_j = q$. Then $B\mathcal{R}B$, and therefore due to the second property $A\mathcal{R}B$.*

Hence, in both cases we have $A\mathcal{R}B$. This concludes the proof.

2.5 Closure under supported formulas

In propositional argumentation, we may have, for example, $\langle \{p \wedge q\}, p \rangle$ as an argument, without having $\langle \{p \wedge q\}, q \rangle$ as an argument. However, in some cases the set of arguments of an extended propositional theory has been generated in some way. In those cases, we may want to consider some closure conditions.

Definition 9 *Let $\langle \mathcal{A}, sup, con \rangle$ be an extended propositional argumentation framework.*

- *A set of arguments $S \subseteq \mathcal{A}$ is said to be* closed under supported formulas *iff for all arguments $A \in \mathcal{S}$ and all propositional formulas p logically implied by $sup(A)$ but not by a strict subset of $sup(A)$, there is an argument $B \in \mathcal{S}$ such that $sup(B) = sup(A)$ and $con(B) = p$.*
- *We say that $\langle \mathcal{A}, sup, con \rangle$ is closed under supported formulas if $\mathcal{A}$ is.*

If an argumentation framework is closed under supported formulas, then its complete extensions are closed as well.

Lemma 1. *If the argument $\langle H, h \rangle$ is in a complete extension, then $\langle H, \wedge H \rangle$ is also in the extension.*

Lemma 2. *If the argument $\langle H, \wedge H\rangle$ is in a complete extension, then $\langle H, h\rangle$ is in the extension for all h logically implied by H, but not by a strict subset of H.*

Theorem 4 *If an extended propositional argumentation framework is closed under supported formulas, then its complete extensions are closed under them as well.*

A consequence of Theorem 4 is that under closure of supported formulas, extended propositional argumentation framework can be reduced to propositional argumentation framework. Consequently, under this assumption, we have again a collapse of the argumentation semantics.

Example 3. Suppose we have an argument $\langle\{p \wedge q\}, p\rangle$ in a complete extension. Then the lemmas show that we have also $\langle\{p \wedge q\}, p \wedge q\rangle$ (lemma 1) and thus $\langle\{p \wedge q\}, q\rangle$ (lemma 2). But then, clearly, there is no use of arguments like $\langle\{p \wedge q\}, p\rangle$. We can restrict ourselves to arguments $\langle\{p \wedge q\}, p \wedge q\rangle$ (under the assumption of closure of suported formulas, of course).

Thus, under the assumption of closure under supported formulas, the distinction between support and conclusion is just syntactic sugar.

3 Preferences among arguments

In this section we consider the extension of symmetric argumentation with preferences. We start with some definitions concerning preferences.

Definition 10 *A (partial) pre-order on a set $\mathcal{A}$, denoted $\succeq$, is a reflexive and transitive relation. $\succeq$ is said to be total if for all $A, B \in \mathcal{A}$ we have $A \succeq B$ or $B \succeq A$. $\succ$ denotes the strict order associated with $\succeq$, i.e., $A \succ B$ iff $A \succeq B$ and not $B \succeq A$.*

A preference relation on $\mathcal{A}$ is a pre-order $\succeq$ on $\mathcal{A}$ such that $\forall A, B \in \mathcal{A}$, $A \succeq B$ (resp. $A \succ B$) expresses that A is at least as preferred as (resp. strictly preferred to) B.

The new preference-based argumentation framework uses an incompatibility and a preference relation. The incompatibility relation should not be interpreted as an attack relation, since incompatibility relations are always symmetric, while attack relations are often asymmetric.

Definition 11 (Incompatibility+preference argumentation framework) *An incompatibility+preference argumentation framework is a triplet $\langle\mathcal{A}, \mathcal{C}, \succeq\rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{C}$ is a symmetric binary incompatibility relation on $\mathcal{A} \times \mathcal{A}$, and $\succeq$ is a preference relation on $\mathcal{A} \times \mathcal{A}$.*

Starting with a set of arguments, a symmetric incompatibility relation, and a preference relation, we exploit the latter two for specifying a Dung-style attack relation. Then we use an arbitrary semantics of Dung to characterize the set of acceptable arguments. In contrast to most other approaches [2, 13] (but see [4, 5] for exceptions), our approach to reasoning about preferences in argumentation does not refer to the internal structure of the arguments. The use of a symmetric incompatibility relation makes sense in many applications such as dialogue when the internal structure of the arguments is not known and thus does not allow to know whether the attack relation is undercut or rebut. Instead we may only know that the two arguments cannot be used together, i.e., they are incompatible.

Definition 12 *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework and $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ an incompatibility+preference argumentation framework. We say that $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents $\langle \mathcal{A}, \mathcal{R} \rangle$ iff for all arguments A and B of $\mathcal{A}$, we have $A \ \mathcal{R} \ B$ iff $A \ \mathcal{C} \ B$ and not $B \succ A$. We say also that $\mathcal{R}$ is represented by $\mathcal{C}$ and $\succeq$.*

Example 4. Let $\langle \{A, B\}, \{A\mathcal{C}B, B\mathcal{C}A\}, \{A \succeq B\} \rangle$. A attacks B but not vice versa.

Since the attack relation is defined from the incompatibility and the preference relation, the other notions introduced by Dung can be applied also to the incompatibility+preference argumentation framework. For example, to determine the grounded semantics of the incompatibility+preference framework, we first compute the attack relation, and then the grounded semantics as in the general case.

An acyclic/symmetric argumentation framework is an argumentation framework in which the attack relation is acyclic/symmetric, *etc*. In this paper we define an acyclic strict attack relation as follows. If there is a strict attack path where argument A_1 attacks argument A_2 but not vice versa, argument A_2 attacks argument A_3 but not vice versa, $\ldots$, then argument A_n does not attack argument A_1.

Definition 13 (Acyclic argumentation framework) *An argument A strictly attacks B if A attacks B and B does not attack A. A strict acyclic argumentation framework is an argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ in which there is no sequence of arguments $\langle A_1, \ldots, A_n \rangle$ such that A_1 strictly attacks A_2, A_2 strictly attacks A_3, ..., A_{n-1} strictly attacks A_n, and A_n attacks A_1.*

To prove that acyclic attacks can be characterized by incompatibilities and preferences, we have to show that the implication holds in both ways. We start with the implication from right to left.

Lemma 3. *If the incompatibility+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents the argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$, then $\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework (in the sense of Definition 13).*

Proof. We prove the lemma by contradiction. Assume there exists an incompatibility+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ representing argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ such that $\langle \mathcal{A}, \mathcal{R} \rangle$ is not a strictly acyclic argumentation framework. In other words, there exists a sequence of strictly attacking arguments $\langle A_1, \ldots, A_n \rangle$ with $A_n \mathcal{R} A_1$. Since $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ represents $\langle \mathcal{A}, \mathcal{R} \rangle$, we have $A_i \ \mathcal{C} \ A_{i+1}$ and not $A_{i+1} \succ A_i$. Due to symmetry of $\mathcal{C}$, we have also $A_{i+1} \ \mathcal{C} \ A_i$. Since the attacks are strict, we do not have not $A_i \succ A_{i+1}$, and we thus have $A_i \succ A_{i+1}$. Moreover, due to transitivity of $\succeq$, we have $A_1 \succ A_n$. This implies that we cannot have $A_n \mathcal{R} A_1$. Contradiction, thus the lemma holds.

Now we show that the implication from left to right holds. We prove this lemma by construction: given an acyclic argumentation framework, we construct an incompatibility+preference framework representing it.

Lemma 4. *If $\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework, then there is an incompatibility+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ that represents it.*

Proof. *By construction. Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be a strictly acyclic argumentation framework. Moreover, consider an incompatibility+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ defined as follows:*

- *$\mathcal{C} = \{(A, B) \mid A\mathcal{R}B$ or $B\mathcal{R}A\}$ is the symmetric closure of $\mathcal{R}$*
- *$\succeq$ is the transitive and reflexive closure of the strict attack relations in $\mathcal{R}$, i.e., the transitive and reflexive closure of $\{(A, B) \mid A\mathcal{R}B$ and not $B\mathcal{R}A\}$*

We show that $\forall A, B \in \mathcal{A}$, $A\mathcal{R}B$ if and only if $A\mathcal{C}B$ and not $B \succ A$. From left to right, we show that $\forall A, B \in \mathcal{A}$ if $A\mathcal{R}B$ then $A\mathcal{C}B$ and not $B \succ A$. Suppose that $A\mathcal{R}B$ and not $A\mathcal{C}B$ or $B \succ A$. By construction we have $A\mathcal{C}B$ due to $A\mathcal{R}B$, and therefore we have $B \succ A$. By construction $B \succ A$ means that there is a sequence of strict attacks $B\mathcal{R}A_1$, $A_1\mathcal{R}A_2$, ..., $A_n\mathcal{R}A$. Consequently, due to the acyclicity property, we do not have $A\mathcal{R}B$. Contradiction.

From right to left, we show that $\forall A, B \in \mathcal{A}$ if $A\mathcal{C}B$ and not $B \succ A$ then $A\mathcal{R}B$. Suppose now that $A\mathcal{C}B$, not $B \succ A$ and not $A\mathcal{R}B$. By construction $A\mathcal{C}B$ means that either $A\mathcal{R}B$ or $B\mathcal{R}A$ holds. Since $A\mathcal{R}B$ does not hold by hypothesis we have $B\mathcal{R}A$. Also by construction $B\mathcal{R}A$ and not $A\mathcal{R}B$ implies that $B \succeq A$. $B \succeq A$ and not $B \succ A$ implies $A \succeq B$. Since $A \succeq B$ and not $A\mathcal{R}B$, $A \succeq B$ must be added by reflexive or transitive closure, and therefore there must be a sequence of strict attacks $A\mathcal{R}A_1$, $A_1\mathcal{R}A_2$, ..., $A_n\mathcal{R}B$. Consequently, due to the acyclicity property, we do not have $B\mathcal{R}A$. Contradiction.

Summarizing, strictly acyclic argumentation frameworks are characterized by incompatibility+preference argumentation frameworks.

Theorem 5 *$\langle \mathcal{A}, \mathcal{R} \rangle$ is a strictly acyclic argumentation framework (in the sense of Definition 13) if and only if there is an incompatibility+preference argumentation framework $\langle \mathcal{A}, \mathcal{C}, \succeq \rangle$ that represents it (in the sense of Definition 12).*

4 Related work

Further developments of Dung's framework have been studied along various directions:

- Dung's abstract framework has been used mainly in combination with more detailed notions of arguments, for example arguments consisting of rules, or arguments consisting of a justification and a conclusion.
- Analogously, various kinds of attack relations have been distinguished, such as rebutting and undercutting.
- Constraints have been imposed on the attack relations, such as symmetry in symmetric argumentation frameworks [9].
- There have been several attempts to modify or generalize Dung's framework, for example by introducing preferences [2], priorities [13], values [5], or collective arguments [8].

We believe that our approach of characterizing argumentation frameworks with properties of Dung's attack relation is a powerful way to relate these various approaches in argumentation theory. For example, though various authors present their argumentation framework as an extension of Dung's framework, which has the technical consequence that they define also new notions of, for example, defence and acceptance, we can consider also their argumentation framework as an alternative representation of Dung's framework, which has the consequence that we do not have to introduce such notions.

The main results of this paper are concerned with preference-based argumentation. Simari and Loui [14] introduce preference relations over arguments, and various proposals have been made how to specify and compute these preferences. The authors of [13, 15] consider arguments composed of defeasible rules, and they use the argument *structure* to derive preference relations. For instance, one argument is more specific about the current evidence than the other one, which makes the first argument stronger. Alternatively, several authors [6, 3, 1] have built arguments from beliefs tagged with explicit priorities, such as certainty levels. The arguments using higher-level beliefs are considered stronger than those using lower-level beliefs. Bench-Capon [5] does not consider the structure of arguments but derives a preference ordering from the values they promote. Since arguments promote only a single value, an argument is better than another one if and only if the value promoted by the former is preferred to the value promoted by the latter argument.

In [11] we consider a representation of Dung's framework by an incompatibility and preference argumentation framework, where A attacks B if and only if A and B are incompatible, and A is at least as preferred as B. The acyclicity or loop condition that characterizes this kind of argumentation framework is that if there is a cycle in the sense that A_1 attacks A_2, ..., A_{n-1} attacks A_n, A_n attacks A_1, then we have that A_2 attacks $A_1, \ldots, A_n$ attacks A_{n-1}, A_1 attacks A_n. The representation used in this paper is much more widely used, and the loop condition of this paper looks more natural than the condition of [11].

5 Summary

Dung's abstract argumentation framework [10] is based only on sets of arguments carrying a binary attack relation. In this paper we develop an abstract framework for incompatible arguments within Dung's abstract argumentation framework. As an example we introduce an instance of Dung's framework where arguments are represented by propositional formulas and an argument attacks another one when the conjunction of their representations is inconsistent, which we characterize as a kind of symmetric attack. Since symmetric attack is known to have the drawback to collapse the various argumentation semantics, we consider two variations.

First, we consider propositional arguments distinguishing support and conclusion, and consider the question whether the collapse of the argumentation semantics be avoided when we represent arguments by pairs $\langle H, h\rangle$, where H is a set of propositional formulas supporting the formula h (i.e., H logically implies h in propositional logic), a kind of propositional argumentation introduced by Amgoud and Cayrol [2]. We show it is nearly as expressive as Dung's framework. However, we show also that when we extend it with a property which we call closure under supported formulas, then a similar collapse arises.

Second, we consider an argumentation framework with an incompatibility relation together with a preference relation over arguments, and the mapping that argument A attacks argument B if and only if A and B are incompatible, and B is not preferred to A, as suggested by Amgoud and Cayrol [2] and Bench-Capon [5]. We characterize the attack relation by a particular kind of loop condition. If there is a sequence of strict attacks $\langle A_1, \ldots, A_n \rangle$, i.e., A_1 attacks A_2 but not vice versa, A_2 attacks A_3 but not vice versa, etc, then A_n does not attack A_1.

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