Game Specification in Normative Multiagent System: the Trias Politica

Guido Boella
Dipartimento di Informatica
Università di Torino - Italy
e-mail: guido@di.unito.it

Leendert van der Torre
SEN-3 - CWI
Amsterdam - The Netherlands
e-mail: torre@cwi.nl

Abstract

In this paper we formalize the specification of games in the trias politica using Rao and Georgeff's specification language BDI$_{CTL^*}$. In particular, we generalize Rao and Georgeff's specification of single agent decision trees to multiagent games, for which we introduce observations and recursive modelling, in this setting we formalize obligations, and we characterize four kinds of agents, called legislators, judges, policemen and citizens. Legislators are characterized by their power to create and revise obligations, judges are characterized by their power to count behavior of citizens as violations, and policemen are characterized by their ability to sanction behavior.

1 Introduction

Montesquieu's trias politica distinguishes between three autonomous powers. Due to their autonomy, these powers can be analyzed as agents and mental attitudes can be attributed to them. In this paper we call these agents legislators, judges and policemen. Moreover, our multiagent model contains a set of agents called citizens. Various games with obligations can be played in this multiagent system. E.g., possible decision problems are:

- A citizen considers whether it will fulfill or violate the obligations, given its expectations of judges and policemen [2, 4].
- A legislator considers which sanctions it associates with norms, such that citizens will fulfill obligations [5, 8].

We are interested in the specification of such decision problems in the trias politica. To take the limited rationality of the agents into account we use recursive modelling [9] instead of equilibria analysis, the most popular approach in game theory. We use Rao and Georgeff's BDI$_{CTL^*}$ and generalize their translation to BDI$_{CTL^*}$ models of single agent decision trees [11] to multiagent games. In this approach, belief accessible worlds represent possible alternatives, desire (or goal) accessible worlds represent the alternatives the agent takes into consideration, and intention accessible worlds represent the optimal decisions. Our research question breaks down into the following questions.

1. How to formally specify recursive games in BDI$_{CTL^*}$?
2. How to formally specify obligations in BDI$_{CTL^*}$?
3. How to characterize legislators, judges and policemen in BDI$_{CTL^*}$?

The motivation of our research questions is the specification of decision problems of the kind introduced by Boella and Lesmo [2]. Normative systems that control and regulate behavior like legal, moral or security systems are autonomous, they react to changes in their environment, and they are pro-active. For example, the process of deciding whether behavior counts as a violation is an autonomous activity. Since these properties have been identified as the properties of autonomous or intelligent agents [15], normative systems may be called *normative agents*. This goes beyond the observation that a normative system may contain agents, like a legal system contains legislators, judges and policemen, because *a normative system itself is called an agent*. The first advantage of the normative systems as agents perspective is that the interaction between an agent and the normative system which creates and controls its obligations can be modelled as a game between two agents. The second advantage of the normative systems as agents perspective is that, since mental attitudes can be attributed to agents, we can attribute mental attitudes to normative systems. The logic proposed in this paper can be used to formally specify games defined in [2, 3, 4, 7] at a high level of abstraction.

Rao and Georgeff do not present a full axiomatization of the formalization of decision trees in their logic. For our much more complicated system we focus also on semantics and we do not present a full axiomatization. However, we do show some interesting properties which characterize important mechanisms of the logic. For example, the characteristic property of recursive modelling is that for each

agent the decision of later agents is fixed.

The layout of this paper is as follows. In Section 2 we discuss Rao and Georgeff's BDI$_{\text{CTL}^*}$. In Section 3 we define recursive decision problems in the logic. In Section 4 we define obligations and we characterize legislators, judges and policemen.

2 The logic

In this section we use an equivalent reformulation of Rao and Georgeff's formalism [12] presented by Schild [13]. We only consider the semantics. Following Rao and Georgeff we do not use desires but goals, because that seems to better fit the interpretation of games.

Definition 1 *Assume n agents. The admissible formulae of BDI$_{CTL^*}$ are categorized into two classes, state formulae and path formulae.*

S1 *Each primitive proposition is a state formula.*
S2 *If α and β are state formulae, then so are $\alpha \wedge \beta$, $\neg\alpha$.*
S3 *If α is a path formula, then $E\alpha$, $A\alpha$ are state formulae.*
S4 *If α is a state formula and $1 \leq i \leq n$, then $B_i(\alpha), G_i(\alpha), I_i(\alpha)$ are state formulae as well.*
P1 *Each state formula is also a path formula.*
P2 *If α and β are path formulae, then so are $\alpha \wedge \beta$, $\neg\alpha$.*
P3 *If α and β are path formulae, then so are $X\alpha$, $\alpha U \beta$.*

The unary operator $\Diamond$ (eventually) is defined as a special case of the binary U operator by $\Diamond\alpha = \top U \alpha$, while $\Box$ (always) is the dual of $\Diamond$ by $\Box\alpha = \neg\Diamond\neg\alpha$. Finally, $\wedge$ and $\rightarrow$ are defined as usual.

The semantics of BDI$_{\text{CTL}^*}$ involves a modal and a temporal dimension. The truth of a formula depends on both the possible world w and the temporal state s. A pair $\langle w, s\rangle$ is called a situation in which BDI$_{\text{CTL}^*}$ formulae are evaluated. The relation between situations is traditionally called an accessibility relation (for beliefs) or a successor relation (for time).

Definition 2 *Assume n agents. A Kripke structure $M = \langle \Delta, \mathcal{R}, \mathcal{B}_1, \mathcal{G}_1, \mathcal{I}_1, \ldots, \mathcal{I}_n, L\rangle$ forms a situation structure if Δ is a set of situations, $\mathcal{R} \subseteq \Delta \times \Delta$ is a binary relation such that $w = w'$ whenever $\langle w, s\rangle \mathcal{R} \langle w', s'\rangle$, $Z_i \subseteq \Delta \times \Delta$ for $Z \in \{B, G, I\}$ and $1 \leq i \leq n$ are binary relations such that $s = s'$ whenever $\langle w, s\rangle Z_i \langle w', s'\rangle$, and L an interpretation function that assigns a particular set of situations to each primitive proposition. $L(p)$ contains all those situations in which p holds.*

A speciality of CTL* is that some formulae – called path formulae– are not interpreted relative to a particular situation. What is relevant here are full paths. The reference to M is omitted whenever it is understood.

Definition 3 *Assume n agents. A full path in situation structure M is an infinite sequence $\chi = \delta_0, \delta_1, \delta_2, \ldots$ such that for every $i \geq 0$, δ_i is an element of Δ and $\delta_i \mathcal{R} \delta_{i+1}$. We say that a full path starts at δ iff $\delta_0 = \delta$. We use the following convention. If $\chi = \delta_0, \delta_1, \delta_2, \ldots$ is a full path in M, then χ^i $(i \geq 0)$ denotes exactly the same infinite sequence as χ, except that the first i components are omitted.*

Let M be a situation structure, δ a situation, and χ a full path. The semantic relation $\models$ for BDI$_{CTL^*}$ is then defined as follows:

S1 $\delta \models p$ *iff* $\delta \in L(p)$.
S2 $\delta \models \alpha \wedge \beta$ *iff* $\delta \models \alpha$ *and* $\delta \models \beta$.
$\delta \models \neg\alpha$ *iff* $\delta \models \alpha$ *does not hold.*
S3 $\delta \models E\alpha$ *iff there is a full path χ in M starting at δ such that $\chi \models \alpha$.*
$\delta \models A\alpha$ *iff for every full path χ in M starting at δ, $\chi \models \alpha$.*
S4 $\delta \models B_i(\alpha)$ *iff for every $\delta' \in \Delta$ such that $\delta \mathcal{B} \delta'$, $\delta' \models \alpha$.*
$\delta \models G_i(\alpha)$ *iff for every $\delta' \in \Delta$ such that $\delta \mathcal{G} \delta'$, $\delta' \models \alpha$.*
$\delta \models I_i(\alpha)$ *iff for every $\delta' \in \Delta$ such that $\delta \mathcal{I} \delta'$, $\delta' \models \alpha$.*
P1 *If α is a state formula and χ starts at δ, then $\chi \models \alpha$ iff $\delta \models \alpha$.*
P2 $\chi \models \alpha \wedge \beta$ *iff* $\chi \models \alpha$ *and* $\chi \models \beta$.
$\chi \models \neg\alpha$ *iff* $\chi \models \alpha$ *does not hold.*
P3 $\chi \models X\alpha$ *iff* $\chi^1 \models \alpha$.
$\chi \models \alpha U \beta$ *iff there is* $i \geq 0$ *with* $\chi^i \models \beta$ *and for all* j, $(0 \leq j < i), \chi^j \models \alpha$.

BDI$_{\text{CTL}^*}$ is very general. In particular, we have epistemic states over temporal sequences (e.g., $B_i A\Box p$, i believes that always p) which is called internal dynamics, and temporal sequences over epistemic states (e.g., $A\Box B_i p$, Always $B_i p$ will be the case) which is called external dynamics. Rao and Georgeff discuss realism properties expressing the fact that for each believed world there must be a goal world which is a subtree of it (and analogously for goals and intentions):

$B_i p \rightarrow G_i p$ $\qquad$ $G_i p \rightarrow \neg B_i \neg p$
$G_i E\Diamond p \rightarrow B_i E\Diamond p$ $\qquad$ $I_i E\Diamond p \rightarrow G_i E\Diamond p$

and commitment strategies (see [12]).

3 Specifying games

In this section we consider the specification of games, based on observations and recursive modelling. Our approach extends Rao and Georgeff's translation of decision trees to BDI$_{\text{CTL}^*}$. We take their decision problems as our starting point.

Rao and Georgeff [11] use an extension of BDI$_{\text{CTL}^*}$ with probabilities and utilities to model single agent decision trees. This is done in the following way:

- Alternatives of decisions are modelled as branches in the branching time logic CTL.

- Beliefs of a decision maker model uncertainty. The translation of decision trees to $\text{BDI}_{\text{CTL}^*}$ models encodes all uncertainty as uncertainty about the actual world, such that given an actual world all actions are deterministic (a well known translation of indeterministic effects in decision theory).
- Goals of a decision maker are the alternatives the agent takes into consideration.
- Intentions of a decision maker are the optimal alternatives.

In this paper we remain faithful to the qualitative setting of BDI logic such that we do not use the probabilities and utilities. We represent the distinction between events and facts by introducing in the logical language a distinction between decision variables and parameters [10], also called controllable and uncontrollable propositions.decision variable in our approach corresponds to the proposition $done(e)$ in Rao and Georgeff's approach, where e is an event. We thus identify decisions with attributing true to decision variables. Joint actions like lifting table can be modelled by two individual decision variables a_1 and a_2 and believed consequences $a_1 \wedge a_2 \rightarrow p$, for p is lifting table.

Definition 4 *Assume n agents. The controllability of the variables is a partitioning of the propositional variables in $A_1, \ldots, A_n, P$, where A_i are controllable propositions of agent i, and P the propositions which are not directly controlled by a single agent.*

We keep the interpretation of belief, goal and intention worlds. The additional concept we introduce is that beliefs and goals about other agents are used to model the various layers of recursive decision models. Beliefs thus refer to other agents' decision models.

In our running example of a recursive decision problem, first agent 0, a legislator, decides which norm to create, then agent 1, a citizen, decides whether or not to fulfill this and other norms, then agent 2, a judge, decides whether or not the behavior of the citizen counts as a violation, and finally agent 3, a policeman, decides whether or not to sanction the behavior. In such recursive decision problems, each agent has to simulate the decision problem of each agent making a decision later than itself. The judge thus has to model the decision problem of the policeman, the citizen has to model the decision problem of the judge and the policeman, and the legislator has to model the decision making of the other three agents. Each of these decision models has to be solved a number of times. For example, the legislator has to solve the citizen's decision problem for each of its own decisions, it has to solve the decision problem of the judge for each possible decision of the legislator and the citizen, *et cetera*. Consequently, in recursive modelling an agent has to solve a large number of decision problems, before it can consider its own decisions. This leads to the problem of finding optimal decisions efficiently, the search for good heuristics, *et cetera*. However, for the specification problem considered in this paper, such efficiency considerations are less relevant.

In Figure 1 we illustrate the subgame between agent 1 which can play x or $\neg x$ and agent 2 which can reply with v or $\neg v$. Moreover, a parameter p represents different circumstances in the alternative worlds. Given the actual world w, the beliefs of agent 1 in state s_0 are represented by worlds w_0 and w_1 starting at s_0. Agent 1 considers that either x or $\neg x$, becasue w_2 and w_3 satisfy in s_0 $G_1(EXx)$ and $G_1(EX\neg x)$. It has the goal that x is followed by v, because this is the result of the recursive modelling of agent 2's behavior in w_{11}. Agent 1 may not like v, but this is not represented.

Note that the intention attributed to agent 2 by agent 1 before its decision is different from after the decision. The initial goal of agent 2 in world w_7 is that x is false and it is followed by $\neg v$. However, after x in w_{10}, since x is observable by agent 2 (we have $B_1(AX(x \rightarrow B_2(x)))$), it changes its mind in world w_{11} and intends to reply with v.

The first assumption of recursive modelling is that we do not consider simultaneous or parallel decisions, because a recursive decision problem contains a sequence of agents who make a decision. In our formalization, such simultaneous decisions may be belief accessible because the decisions may be possible, but they are not goal accessible because they will not be taken into account. This is characterized by the following axiom. For each two distinct agents i and j, i.e., $i \neq j$, for each proposition d_1 built from A_i and d_2 built from A_j (such that d_1 and d_2 are not tautologies):
$G_i d_1 \rightarrow G_j \neg d_2$

The second assumption which we make here is that that each agent only makes a single decision in the decision problem. This is characterized by the following property for any two decision variables p_1 and p_2 of the same agent j, i.e., $p_1, p_2 \in A_j$, and for any sequence $AXAX\ldots$, which we write as $(AX)^n$.
$G_i p_1 \rightarrow G_i (AX)^n \neg p_2$ for $n > 0$

To model such a sequence of decisions, we have to encode two things. First, the crucial mechanism for recursive modelling is that an agent assumes that each recursively modelled other agent's decisions are fixed for it. So, in our running example for agent 0 the decisions of agent 1, 2 and 3 are fixed for it, for agent 1 the decisions of agent 2 and 3 are fixed, and for agent 2 the decisions of agent 3 are fixed. The notion of fixation can now be characterized as follows. If $j > i$ and α is built from A_j, then:
$B_i(I_j(EX\alpha)) \rightarrow G_i(EX\alpha) \quad B_i(I_j(AX\alpha)) \rightarrow G_i(AX\alpha)$

For example, in the model in Figure 1, we have due to the axioms above that $B_1(AX(x \rightarrow I_2(AXv))$, since $\langle w_{11}, s_1 \rangle$ is the only accessible situation from all the be-

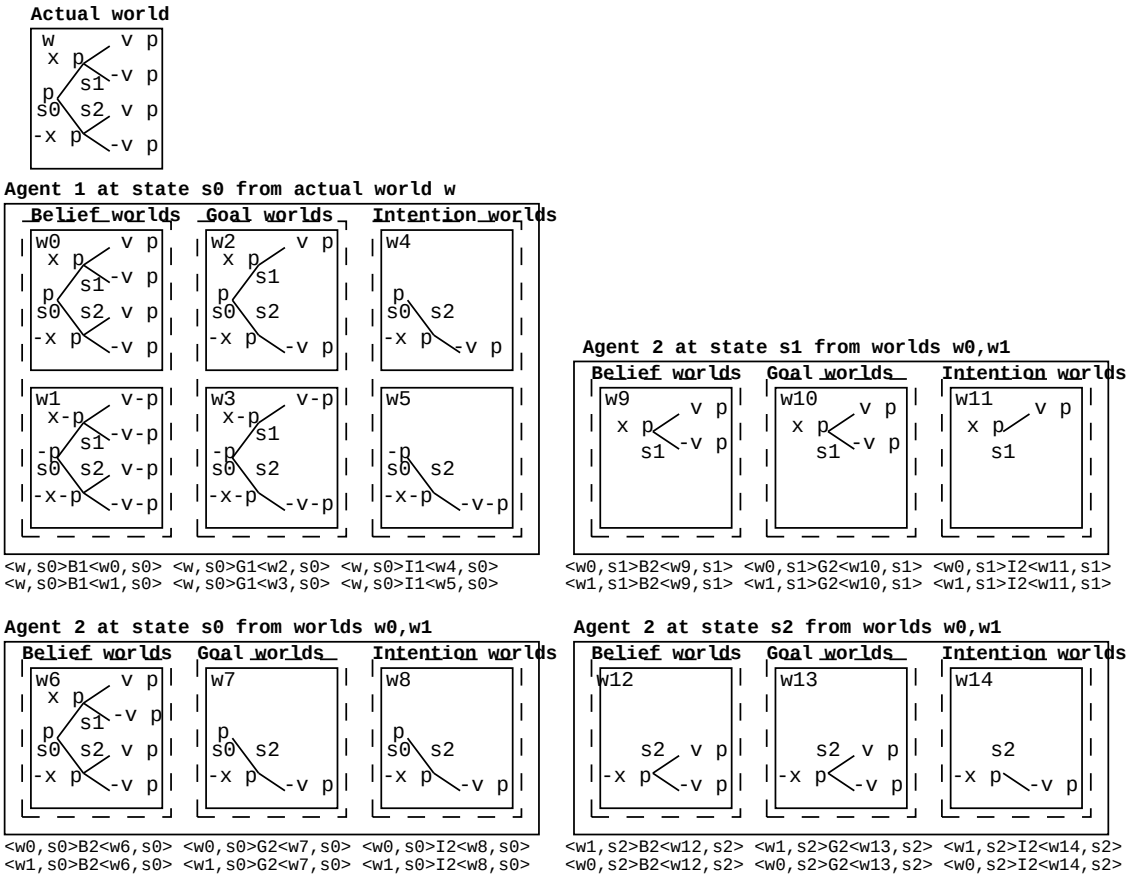

Figure 1. Recursive modelling.

lieved situations $\langle w_0, s_1\rangle$ and $\langle w_1, s_1\rangle$.

Second, when an agent is not making a decision, it may be influenced by the decisions of other agents. This is done by observing the decision or its effects. For example, the initial state of the agent 2's decision is its initial state, updated by observations from agent 1's decision. We do not formalize sensing actions.

Definition 5 *The set of observable atoms of agent i is a subset of the propositional variables. We say that $Ob_i(p)$ is true if p is built from observable atoms of agent i.*

The characteristic property of observations is that if an agent i decides something, then another agent j can observe it and assume it as fixed for its decision making.
$Ob_j(p) \rightarrow B_i(AX(p \rightarrow B_j(p)))$

The models may also satisfy a version of the above fixation axioms for beliefs, which is related to a notion of realism which can be adopted for other agents beliefs. We call this property transparency of decision variables: every agent considers possible the alternatives at disposal of the other agents. If α is built from A_j, then:

$B_i(B_j(EX\alpha)) \rightarrow B_i(EX\alpha)$
$B_i(B_j(AX\alpha)) \rightarrow B_i(AX\alpha)$

For example, in Figure 1 agent 1 in w_0 and w_1 believes that the options of agent 2 are v and $\neg v$, because agent 1 believes that agent 2 that it has these options in w_9 and w_{12}.

4 Obligations and characterizing agents

We formalize obligations in this setting. It is inspired by the so-called Anderson's reduction of deontic logic to alethic logic [1], which may be written as $O(p) = \Box(\neg p \rightarrow V)$. In this definition V is a so called violation constant. To distinguish between violations, we assume that there is a set of them. In particular, we assume that a subset of the propositions represents that a norm of this normative system is violated, see [14] for details.

Definition 6 *The violation set is a subset of the propositional variables. We say that $V(p)$ is true if p is built from the violation set only.*

Likewise, some decision variables are known as sanctions.

Definition 7 *The sanction set is for each agent a subset of the propositional variables. We say that $S_i(p)$ is true if p is built from the sanction set of agent i.*

There are various ways to encode obligations in a BDI setting. Following ideas in [2, 3, 4, 7] we say that an obligation of an agent corresponds to a goal of the normative system. This has been paraphrased by "your wish is my command". Judges and policemen are defined by further obligations. Due to space limitations, we cannot discuss this definition any further.

Definition 8 (Obligations) *Agent i is* obliged *to do a (a decision variable in A_i) with sanction s, represented by $O_{i,j,k,l}(a, s)$, iff there exists agents j, k, l such that:*

1. *$G_j(a)$: agent j (legislator) wants that a;*
2. *$G_j(\neg a \rightarrow AXp)$: agent j wants that absence of a implies p;*
3. *$p \in A_k$ and $V(v)$: v is a decision variable of agent k (judge) which says that behavior counts as a violation;*
4. *$G_j AX(v \rightarrow AXs)$: agent j wants that v implies s;*
5. *$s \in A_l$ and $S_i(s)$: s is a decision variable of agent l (policeman) which represents a sanction for agent i;*
6. *$G_j A\Box\neg v$: agent j desires that there are no violations.*
7. *$G_j A\Box\neg s$: agent j desires that there are no sanctions.*

The spirit of the Montesquieu's *trias politica* is the principle of the separation of powers. Autonomous agent i is respectively:

legislator if it can change the obligations, i.e., there is some $1 \leq a \leq n$ such that we have: $(\neg O_{a,i,k,l}(p, s) \wedge a) \rightarrow AXO_{a,i,k,l}(p, s)$.

judge if it has the power to count behavior of citizens as a violation, i.e., there is a $a \in A_i$ such that $V(a)$.

policeman if it can sanction behavior, i.e., there is a $a \in A_i$ such that $S_j(a)$ for some j.

The separation of powers in the trias politica means that agents cannot be both a legislator and a judge, or both a legislator and a policeman, or both a judge and a policeman. This can be characterized in the obvious way.

5 Concluding remarks

In this paper we formalize the specification of games in the trias politica. We use Rao and Georgeff's specification language $\text{BDI}_{\text{CTL}^*}$. In particular, we generalize Rao and Georgeff's specification of single agent decision trees to multiagent games, for which we introduce observations and recursive modelling. We formalize obligations in this setting, and we distinguish four kinds of agents, called legislators, judges, policemen and citizens. Legislators are characterized as agents that can change the obligations, judges are characterized by their power to count behavior of citizens as a violation, and policemen are characterized by their ability to sanction behavior.

Each set of normative agents can again be ordered in higher and lower order legislators, judges and policemen, which leads to three hierarchies of normative agents. A higher court considers which permissions it can create such that lower courts will not introduce norms the higher court deems undesirable [6]. The specification of such games is subject of further research.

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