An Update Semantics for Deontic Reasoning

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Abstract

In this paper we propose the deontic logic OUS, that formalizes reasoning about prescriptive obligations in update semantics. In OUS the definition of logical validity of obligations is not based on truth values but on action dynamics. You know the meaning of a normative sentence if you know the change it brings about in the betterness relation of anyone who is subjected to the news conveyed by it. We argue that this dynamic interpretation of obligations is important for applications of deontic logic in computer science, because it can be used to analyze the dynamics of obligations.

1 Introduction

One of the first topics discussed in the development of deontic logic was the question whether norms have *truth values*. For example, von Wright was hesitant to call deontic formulas 'logical truths,' because "it seems to be a matter of extra-logical decision when we shall say that 'there are' or 'are not' such and such norms" [vW81]. Alchourrón and Buligyn discussed the *possibility* of a logic of norms, which they distinguish from the logic of normative propositions.

> "One such issue is the problem of the possibility of a logic of norms. Some authors think that there are logical relations between the norms, and so favor the development of a specific logic of norms (sometimes called 'deontic logic', though 'normative logic' would perhaps be a more appropriate name). Other authors deny the very possibility of such a logic because in their view there are no logical relations between norms. According to them deontic logic can only assume the form of a logic of normative propositions, i.e. (true or false) propositions about (the existence of) norms." [AB81]

The distinction between norms and normative propositions is that the former are prescriptive whereas the latter are descriptive. Alchourrón explains the distinction with the following box metaphor.

> "We may depict the difference between the descriptive meaning (normative propositions) and the prescriptive meaning (norm) of deontic sentences by means of thinking the obligatory sets as well as the permitted sets as different boxes ready to be filled. When the authority α uses a deontic sentence prescriptively to norm an action, his activity belongs to the same category as *putting something into a box.* When α, or someone else, uses the deontic sentence descriptively his activity belongs to the same category as *making a picture of* α *putting something into a box.* A proposition is like a picture of reality, so to assert a proposition is like making a picture of reality. On the other hand to issue (enact) a norm is like putting something in a box. It is a way of creating something, of building a part of reality (the normative qualification of an action) with the purpose that the addressees have the option to perform the authorized actions while performing the commanded actions." [Alc93]

From Alchourrón's box metaphor follows that prescriptive obligations are interpreted *dynamically* whereas descriptive obligations are interpreted *statically.* A logic of prescriptive obligations – a logic of norms, a logic without truth values – can therefore be used to analyze the dynamics of obligations. Whereas descriptive deontic logics only formalize reasoning about obligations – which obligations follow from a set of obligations – prescriptive obligations also reason with obligations. This makes such a logic extremely important for applications of deontic logic in computer science. For example, consider contractual obligations in trade scenarios. Obligations are created as the result of contractual agreements, and later discharged when they are either fulfilled or violated. Prescriptive obligations are suited to formalize this reasoning with obligations.

In this paper we introduce an update semantics to formalize prescriptive obligations. This update semantics is based on the update semantics of Veltman [Vel96]. We call the resulting system OUS for Obligations in Update Semantics. In the standard definition of logical validity, an argument is valid if its premises cannot all be true without its conclusion being true as well. In update semantics, the slogan 'you know the meaning of a sentence if you know the conditions under which it is true' is replaced by 'you know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it.' Thus meaning becomes a dynamic notion: the meaning of a sentence is an operation on information states. In this paper, we propose the following definition of meaning for normative statements. The information states of update semantics are instantiated with betterness relations, i.e. preference orderings reflecting different degrees of ideality [Han71, Lew74].

> You know the meaning of a normative sentence if you know the change it brings about in the betterness relation of anyone who is subjected to the news conveyed by it.

This dynamic interpretation of prescriptive obligations is related to Alchourrón's box metaphor. Alchourrón compares a (monadic) prescriptive obligation with the *action* of putting something in a box. In our proposal a (dyadic) prescriptive obligation is compared with an *action* of creating an ordering. Updating betterness relations is based on the following two principles.

1. The initial state is an ordering in which all worlds are equivalent.

2. Adding obligations is deleting preference relations between worlds.

A betterness relation can be represented by a possible worlds model, i.e. a set of worlds with a binary relation and a valuation function. We propose to formalize the update of a betterness relation by the obligation 'α ought to be (done) if β is (done)' with the action of deleting all relations between worlds w_1 and w_2 (i.e. $w_1 \leq w_2$) where w_1 satisfies $\neg\alpha \wedge \beta$ and w_2 satisfies $\alpha \wedge \beta$. For example, consider the update of the initial state by the obligation 'p ought to be (done)' represented in Figure 1. This figure should be read as follows. A box represents a betterness relation. A circle is an equivalence class of worlds, satisfying the propositions written within the circle. The symbol $\top$ stands for any tautology, for example $p \vee \neg p$. The left box represent the initial state, the betterness relation in which all worlds are equivalent. The right box represent the betterness relation that is created by updating the initial state with the obligation 'p ought to be (done).' By deleting all relations $w_1 \leq w_2$ where w_1 is a $\neg p$ world and w_2 is a p world, we end up with a betterness relation in which each p world is preferred to all the $\neg p$ worlds.

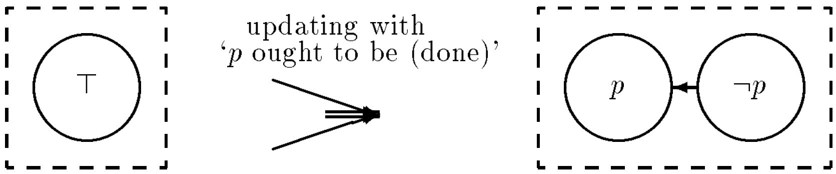

Figure 1: Updating the initial state with 'p ought to be (done)'

For the dynamic interpretation of facts we make the distinction between the context of justification and the context of deliberation explicit in OUS. This distinction originates from temporal deontic logic, where Thomason [Tho81] distinguishes between two ways in which the truth values of deontic sentences are time-dependent. First, these values are time-dependent in the same, familiar way that the truth values of all tensed sentences are time-dependent. Second, their truth values are dependent on a set of choices or future options that varies as a function of time. The distinction between the context of justification and the context of deliberation follows from the second way. The context of deliberation is the set of choices when you are looking for practical advice, and only considers the set of future options. The context of justification is the set of choices for someone who is judging you, and also considers past options which are not realisable anymore. The distinction between the context of justification and the context of deliberation made in temporal deontic logic is not explicitly made in preference-based deontic logic. Instead, there is a rather obscure notion of 'settledness' [Han71]. The antecedent of a dyadic conditional is assumed to be fixed, see for example Hansson's discussion on circumstances. The distinctions 'contexts of deliberation versus justification' and 'settled versus unsettled' are obviously related. The essential contrast between justification and deliberation is a difference in what we take as settled [Tho81]. In this paper we assume that propositional formulas are facts that are settled, and these settled facts describe the context of deliberation. As a consequence, propositional formulas do not describe fluents, because fluents can (by definition) change over time and are thus not settled. For example, we cannot use a propositional variable s for the fluent 'Ron is smoking' but we have to use it as 'Ron is smoking at moment t.' An example of an update with a fact is represented in Figure 2. This figure should be read as follows. A dashed box determines the context of justification and the non-dashed box determines the context of deliberation. The left box represents the betterness relation that is created by updating the initial state with the obligation 'p ought to be (done),' see Figure 1. The right box represents the betterness relation that is created by updating this state with the fact 'p is not (done).' After the fact $\neg p$ has been settled, the context of deliberation has shrunk to the right violation circle.

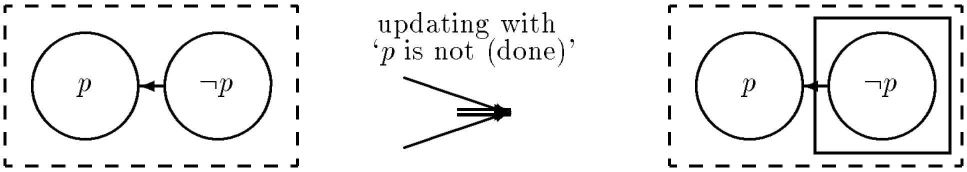

Figure 2: Updating the state created by 'p ought to be (done)' with 'p is not (done)'

We distinguish between prescriptive obligations and tests. In this paper we write $\mathsf{oblige}(\alpha \mid \beta)$ for the prescriptive obligation 'α ought to be (done), if β is (done)' and we write $\mathsf{ideal}(\alpha|\beta)$ for the test 'ideally, α is (done), if β is (done).' To distinguish between the context of justification and the context of deliberation we write oblige^* and ideal^* for the latter. The prescriptive obligations have the dynamic component of creating a new deontic state with a new betterness relation, whereas the tests evaluate what the norms are in a particular deontic state depending on its betterness relation. This is analogous to the difference that Alchourrón meant by his metaphor of the box. Creating norms by 'putting something in the box' is done by the prescriptive obligation $\mathsf{oblige}(\alpha|\beta)$, and evaluating norms by 'making a picture (or reading) of putting something in the box' is done by the test $\mathsf{ideal}(\alpha|\beta)$. For the dynamic interpretation of a test we define it analogously to the test operator in dynamic logic, and to the might and $\mathsf{presumably}$ operators in Veltman's update semantics [Vel96]. If the test is successful then the information conveyed by the test is already subsumed by the information state and the test update simply returns the state. If the test is unsuccessful, then the test update returns the absurd state.

2 Prescriptive obligations

In this section we define prescriptive obligations in update semantics, and we give an example.

2.1 Update semantics

We start with the basic definitions of Veltman's update semantics [Vel96]. To define an update semantics for a language L, one has to specify a set Σ of relevant information states, and a function $[]$ that assigns to each sentence ϕ an operation $[\phi]$ on Σ. If σ is a state and ϕ a sentence, then we write '$\sigma[\phi]$' to denote the result of updating σ with ϕ. We can write '$\sigma[\psi_1]\ldots[\psi_n]$' for the result of updating σ with the sequence of sentences $\psi_1, \ldots, \psi_n$. Moreover, one of the information states has to be labeled as the minimal information state, written as $\mathbf{0}$, and another one as the absurd state, written as $\mathbf{1}$.

Definition 1 (Update system) *An update system is a triple $\langle L, \Sigma, [] \rangle$ consisting of a logical language L, a set of relevant information states Σ and a function $[]$ that assigns to each sentence ϕ of L an operation.*

We discriminate between successful and unsuccessful updates. An update is successful if it does not result in the absurd state, and unsuccessful otherwise.

Definition 2 (Success) *Let σ be an information state and ϕ a formula of L. The update $\sigma[\phi]$ is successful if and only if $\sigma[\phi] \neq \mathbf{1}$.*

A crucial notion of update systems is acceptance. The formula ϕ is accepted in an information state σ, written as $\sigma \Vdash \phi$, if the update by ϕ results in the same state. In that case, the information conveyed by ϕ is already subsumed by σ. If an update is accepted then the information state usually has a specific property. Acceptance is the counterpart of satisfaction in standard semantics.

Definition 3 (Acceptance) *Let σ be an information state and ϕ a formula of L. $\sigma \Vdash \phi$ if and only if $\sigma[\phi] = \sigma$.*

Different notions of validity can be based on the notion of acceptance. We are interested in the following one. An argument is valid if updating the minimal state $\mathsf{0}$ with the premises $\psi_1, \ldots, \psi_n$ in that order, yields an information state in which the conclusion is accepted.

Definition 4 (Validity) *Let $\psi_1, \ldots, \psi_n$ and ϕ be sentences of L. $\psi_1, \ldots, \psi_n \Vdash \phi$ if and only if $\mathsf{0}[\psi_1]\ldots[\psi_n] \Vdash \phi$.*

In the following section we define obligations in update semantics using Veltman's basic definitions.

2.2 Obligations in update semantics

To define obligations in update semantics we have to define the deontic language, the deontic information states and the deontic updates. The deontic language is a propositional language with the dyadic operators $\mathsf{oblige}(\alpha\,|\,\beta)$, $\mathsf{ideal}(\alpha\,|\,\beta)$, $\mathsf{oblige}^*(\alpha\,|\,\beta)$ and $\mathsf{ideal}^*(\alpha\,|\,\beta)$. The operators oblige and oblige^* refer to the obligations as reducing betterness relations, and the operators ideal and ideal^* are used as tests. Finally, the operators oblige and ideal refer to the context of justification and oblige^* and ideal^* refer to the context of deliberation.

Definition 5 (Deontic language) *Let A be a set of atoms and L_0^A a propositional language with A as its non-logical symbols. A string of symbols ϕ is a sentence of L_1^A if and only if either ϕ is a sentence of L_0^A or there are two sentences ψ_1, ψ_2 of L_0^A such that $\phi = \mathsf{oblige}(\psi_1 \mid \psi_2)$, $\phi = \mathsf{ideal}(\psi_1 \mid \psi_2)$, $\phi = \mathsf{oblige}^*(\psi_1 \mid \psi_2)$ or $\phi = \mathsf{ideal}^*(\psi_1 \mid \psi_2)$. We write $\mathsf{oblige}\ \psi$ for $\mathsf{oblige}(\psi \mid \top)$, $\mathsf{ideal}\ \psi$ for $\mathsf{ideal}(\psi \mid \top)$, etc.*

The deontic information states are betterness relations which we represent by possible worlds models $\langle W, \leq, V\rangle$, i.e. binary relations $\leq$ on sets of words W with a valuation function V for propositions at the worlds. The notion of truth does not play a role in the update semantics. That is, we write $\sigma, w \models \phi$ to denote that world w of σ satisfies the propositional formula ϕ, but we leave $\sigma \models \phi$ undefined! We assume that the binary relation is reflexive, but we do *not* assume that it is transitive or total.

We extend the possible world model with a second betterness relation, which is a subrelation of the first one. The complete relation is used for the context of justification and the subrelation is used for the context of deliberation. Summarizing, whereas in Kripke semantics there is a world singled out, called the actual world, we single out a set of worlds, called the context of deliberation.

Definition 6 (Deontic state) *Let L_1^A be a deontic language. Assume a set of worlds W and a valuation function V for L_0^A such that for every consistent ϕ of L_0^A there is a $w \in W$ such that $\mathbf{0}, w \models \phi$. A deontic state is a tuple $\Sigma = \langle W, W^*, \leq, V\rangle$ consisting of the set of worlds W, a possibly empty subset $W^* \subseteq W$, a reflexive (but not necessarily transitive) binary relation $\leq$ on W and the valuation function V.*

$\mathbf{0}$, the minimal state*, is the state given by $\langle W, W, W \times W, V\rangle$, and*

$\mathbf{1}$, the absurd state*, is the state given by $\langle W, \emptyset, \emptyset, V\rangle$.*

From Definition 5 and 6 follows that we define an update system for a specific A, W and V. We do not mention this any further in this paper but leave it implicitly understood. The deontic updates are operations on the deontic states that either zoom in on the betterness relation (for facts), reduce the betterness relation (for obligations) or test ideal worlds. The prescriptive obligations have the dynamic component of creating a new deontic state with a new betterness relation, whereas the tests evaluate what the norms are in a particular deontic state depending on its betterness relation. We first define the reduction of a betterness relation by an obligation.

Definition 7 (Reduction) *Let $\sigma = \langle W, W^*, \leq, V\rangle$ be a deontic state. The reduction of σ by $\mathsf{oblige}(\alpha|\beta)$ ($\mathsf{oblige}^*(\alpha|\beta)$), denoted by the symbol $-$, is defined as follows.*

$\sigma - \mathsf{oblige}(\alpha \mid \beta) = \langle W, W^*, \leq - \{w_1 \leq w_2 \mid \sigma, w_1 \models \neg\alpha \wedge \beta$ *and* $\sigma, w_2 \models \alpha \wedge \beta$ *and* $w_1, w_2 \in W\}, V\rangle$.

$\sigma - \mathsf{oblige}^*(\alpha \mid \beta) = \langle W, W^*, \leq - \{w_1 \leq w_2 \mid \sigma, w_1 \models \neg\alpha \wedge \beta$ *and* $\sigma, w_2 \models \alpha \wedge \beta$ *and* $w_1, w_2 \in W^*\}, V\rangle$.

Finally we deal with dilemmas. The update $\sigma[\mathsf{oblige}(\alpha \mid \beta)]$ is the reduction of σ by $\mathsf{oblige}(\alpha|\beta)$ if afterwards the preferred β worlds are α worlds. Otherwise there is a dilemma and the result of the update is the absurd state. For this definition of deontic updates we need a test whether the preferred β worlds are α worlds. The complex Definition 8 takes care of infinite descending chains, see [Lew73]. We cannot use the following simpler definition,

> 'there is a world w_1 such that $\sigma, w_1 \models \alpha \wedge \beta$ and for all worlds $w_2 \leq w_1$ we have $\sigma, w_2 \models \beta \rightarrow \alpha$,'

because our orderings are not necessarily strongly connected, i.e. we do not have for all w_1, w_2 that either $w_1 \leq w_2$ or $w_2 \leq w_1$. There can be several preferred equivalence classes of β worlds, and in all the worlds of these classes α has to be true.

Definition 8 (pref) *Let $\sigma = \langle W, W^*, \leq, V\rangle$ be a deontic state. The preferred β worlds of W (W^*) of σ satisfy α, written as $\mathsf{pref}(\sigma,\beta) = \alpha$ ($\mathsf{pref}^*(\sigma,\beta) = \alpha$), if and only if for all worlds w_1 such that $\sigma, w_1 \models \neg\alpha \wedge \beta$, there is a world $w_2 \leq w_1$ such that $\sigma, w_2 \models \alpha \wedge \beta$ and for all worlds $w_3 \leq w_2$ we have $\sigma, w_3 \models \beta \rightarrow \alpha$ with $w_1, w_2, w_3 \in W$ ($w_1, w_2, w_3 \in W^*$).*

Putting it all together we define the updates.

Definition 9 (Deontic updates) *Let $\sigma = \langle W, W^*, \leq, V\rangle$ be a deontic state. The update function $\sigma[\phi]$ is defined as follows.*

- *if ϕ is a factual sentence of L_0^A, then*
  - *if there is a $w \in W^*$ such that $\sigma, w \models \phi$, then $\sigma[\phi] = \langle W, \{w \in W^* \mid \sigma, w \models \phi\}, \leq, V\rangle$;*
  - *otherwise, $\sigma[\phi] = \mathbf{1}$.*
- *if $\phi = \mathsf{oblige}(\alpha|\beta)$, then*
  - *if $\mathsf{pref}(\sigma - \mathsf{oblige}(\alpha|\beta), \beta) = \alpha$ then $\sigma[\phi] = \sigma - \mathsf{oblige}(\alpha|\beta)$;*
  - *otherwise, $\sigma[\phi] = \mathbf{1}$.*
- *if $\phi = \mathsf{ideal}(\alpha|\beta)$, then*
  - *if $\mathsf{pref}(\sigma, \beta) = \alpha$, then $\sigma[\phi] = \sigma$;*
  - *otherwise, $\sigma[\phi] = \mathbf{1}$.*
- *if $\phi = \mathsf{oblige}^*(\alpha|\beta)$, then*
  - *if $\mathsf{pref}^*(\sigma - \mathsf{oblige}^*(\alpha|\beta), \beta) = \alpha$ then $\sigma[\phi] = \sigma - \mathsf{oblige}^*(\alpha|\beta)$;*
  - *otherwise, $\sigma[\phi] = \mathbf{1}$.*
- *if $\phi = \mathsf{ideal}^*(\alpha|\beta)$, then*
  - *if $\mathsf{pref}^*(\sigma, \beta) = \alpha$, then $\sigma[\phi] = \sigma$;*

– *otherwise,* $\sigma[\phi] = \mathbf{1}$.

Having defined the deontic language, the deontic information states and the deontic updates we have defined the deontic update system. In the following section we consider a simple example.

2.3 A simple example

The following example illustrates unconditional obligations in the deontic update system. In particular, it illustrates that obligations are not deductively closed, but they satisfy the conjunction and disjunction rule. Moreover, it illustrates the distinction between the context of justification and the context of deliberation.

Example 1 (Three unrelated obligations) *Consider the deontic states in Figure 3, which we write as* $\sigma_1, \ldots, \sigma_6$ *respectively. This figure should be read as follows. The six dashed boxes are the deontic information states, i.e. the betterness relations. They contain all worlds, i.e. they determine the context of justification. The circles within the boxes are equivalence classes of worlds. For each equivalence class of worlds, we only write the positive atoms below. The non-dashed boxes contains only the worlds of* W*, i.e. the worlds of the context of deliberation. The arrows represent strict preferences between the worlds represented by the circles. We leave the transitive closure implicit. The figure also represents five updates: three updates by obligations* $[\mathsf{oblige}\ p]$, $[\mathsf{oblige}\ q]$ *and* $[\mathsf{oblige}\ r]$*, and two updates by facts* $[p]$ *and* $[\neg q]$*. We have* $\sigma_1 \not\Vdash \mathsf{oblige}\ p$, $\sigma_2 \Vdash \mathsf{oblige}\ p$*, and* $\sigma_2 \not\Vdash \mathsf{oblige}(p \vee q)$*. The latter shows that* oblige *is not closed deductively, i.e. we have (see Definition 4)*

$$\mathsf{oblige}\ p \not\Vdash \mathsf{oblige}(p \vee q)$$

Moreover, we have $\sigma_3 \Vdash \mathsf{oblige}(p \wedge q)$ *and* $\sigma_3 \Vdash \mathsf{oblige}(p \vee q)$*, which shows that the obligations have the conjunction and disjunction rule.*

$$\mathsf{oblige}\ p, \mathsf{oblige}\ q \Vdash \mathsf{oblige}(p \wedge q)$$
$$\mathsf{oblige}\ p, \mathsf{oblige}\ q \Vdash \mathsf{oblige}(p \vee q)$$

The figure also show the distinction between the context of deliberation and the context of justification. The latter two updates illustrate that the context of deliberation shrinks when we add facts, whereas the context of justification remains the same. In the latter deontic state σ_6 *we have that the ideal worlds in the context of justification are* $p \wedge q \wedge r$ *worlds, whereas the optimal worlds in the context of justification are* $p \wedge \neg q \wedge r$ *worlds.*

$$\mathsf{oblige}\ p, \mathsf{oblige}\ q, \mathsf{oblige}\ r, p, \neg q \Vdash \mathsf{ideal}(p \wedge q \wedge r)$$
$$\mathsf{oblige}\ p, \mathsf{oblige}\ q, \mathsf{oblige}\ r, p, \neg q \Vdash \mathsf{ideal}^*(p \wedge \neg q \wedge r)$$

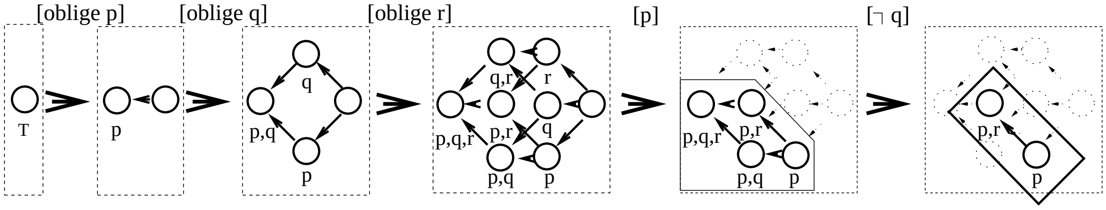

Figure 3: Obligations as updates

A disadvantage of the representation of the three obligations in Example 1 is that we did not represent any temporal information in the three obligations. For example, we did not specify which obligation has to be fulfilled first. In classical deontic logics temporal information is represented implicitly by *conditional* obligations. The following example illustrates how temporal information can be represented by conditional obligations in update semantics.

Example 2 (Three unrelated obligations, continued) *Consider the three conditional obligations* $\mathsf{oblige}\ p$*,* $\mathsf{oblige}(q|p)$ *and* $\mathsf{oblige}(r|p \wedge q)$*. Figure 4 represents the updates of the three obligations and of the facts that p is done and that q is not done. The crucial distinction between this example and the previous one is that there are no obligations left once an obligation has been violated.*

$$\mathsf{oblige}\ p, \mathsf{oblige}\ q, \mathsf{oblige}\ r, p, \neg q \Vdash \mathsf{oblige}^* r$$
$$\mathsf{oblige}\ p, \mathsf{oblige}(q|p), \mathsf{oblige}(r|p \wedge q), p, \neg q \not\Vdash \mathsf{oblige}^* r$$

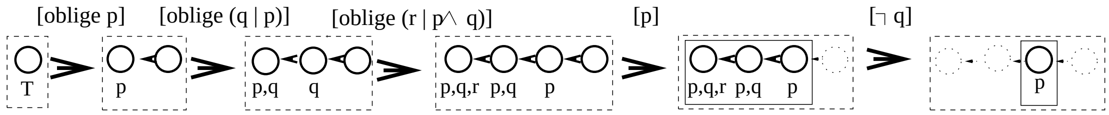

Figure 4: Conditional obligations as updates

Example 1 and 2 illustrate that the number of equivalence classes is a measure of the number of obligations, because the number of equivalence classes increases when we update by obligations. When we compare Figure 3 and 4 we observe that with conditional obligations in Example 2 the set of equivalence classes only increases linearly, whereas with unconditional obligations in Example 1 it increases exponentionally. The number of the equivalence classes of the updates in Figure 3 and 4 are represented in Figure 5. Obviously this attractive feature of conditional obligations is not true in general (if only because absolute obligations are a special type of conditional obligations).

3 Conclusions

In this paper we introduced a logic of prescriptive obligations. All deontic logics known to us are static in the sense that they only formalize reasoning about *descriptive* obligations. They are logics of normative propositions.

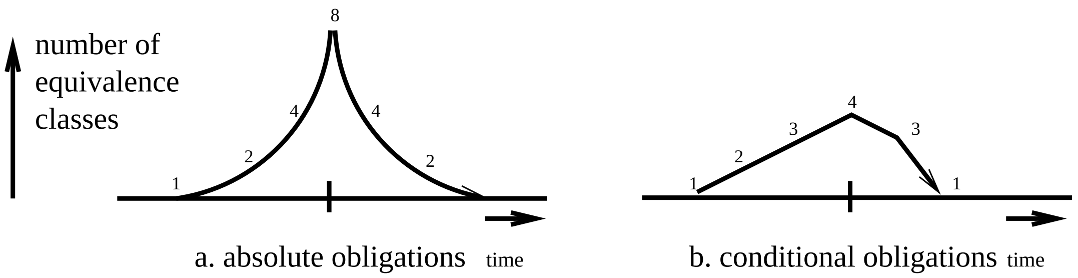

Figure 5: The number of obligations as a function of time

The dynamic approach is probably also a good basis for the development of an efficient proof method for deontic reasoning based on model checking. The idea is first to explicitly construct the deontic state for a set of premises, and then check whether a formula is accepted in this state. This will be investigated in further research.

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