

# Two aggregation paradoxes in social decision making: the Ostrogorski paradox and the discursive dilemma\*

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## Abstract

The Ostrogorski paradox and the discursive dilemma are seemingly unrelated paradoxes of aggregation. The former is discussed in traditional social choice theory, while the latter is at the core of the new literature on judgment aggregation. Both paradoxes arise when, in a group, each individual consistently makes a judgment, or expresses a preference, (in the form of yes or no) over specific propositions, and the collective outcome is in some respect inconsistent. While the result is logically inconsistent in the case of the discursive paradox, it is not stable with respect to the level of aggregation in the case of the Ostrogorski paradox. In the following I argue that, despite these differences, the two problems have a similar structure. My conclusion will be twofold: on the one hand, the similarities between the paradoxes support the claim that these problems should be tackled using the same aggregation procedure; on the other hand, applying the same procedure to these paradoxes will help clarifying the strength and weakness of the aggregation method itself. More specifically, I will show that an operator defined in artificial intelligence to merge belief bases can deal with both paradoxes.

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# 1 Introduction

Consider a group of individuals where each member makes a judgment on several issues. The judgments have a binary form, i.e. each issue can only be accepted (yes, 1) or rejected (no, 0). The issues are not independent, the acceptance or rejection of some propositions entails the acceptance or rejection of other propositions. Suppose now that all the individuals express a judgment on each issue fulfilling the constraints of interdependence between them. Problems arise when the adopted aggregation procedure selects a collective outcome that does not satisfy the interdependencies between the propositions.

This is the case for two paradoxes: the Ostrogorski paradox (Daudt and Rae 1976, Kelly 1989) and the discursive dilemma (Brennan 2001, Kornhauser 1992, Kornhauser and Sager 1986, 1993).<sup>1</sup> The paradoxical character consists in a disturbing conflict between individual and collective rationality: despite the fact that the group members rationally express their judgments (each individual is aware of the interdependence of the propositions and expresses her opinion accordingly), the group outcome – selected by what appears to be a reasonable aggregation procedure – violates these interdependence relations.

While the Ostrogorski paradox is a voting paradox traditionally studied in social choice theory, the discursive dilemma is a new aggregation paradox attracting the interest of scholars in economics, political philosophy, logic and (more recently) social epistemology (Goldman 2004a, List 2005b). Relationships between these two problems and the more familiar paradox of preference aggregation in social choice theory have been investigated. In particular, Bezembinder and van Acker (1985) proved that in every instance of the Ostrogorski paradox there is an underlying Condorcet paradox<sup>2</sup>, and List and Pettit (2002) showed that the discursive paradox is a generalization of the paradox of voting. As in the Condorcet paradox, the collective outcome is (in some respect) inconsistent, despite of the individual inputs being consistent. The result is not stable with respect to the level of aggregation in the Ostrogorski paradox, and it is logically inconsistent in the case of the discursive paradox.

Judgment aggregation is the name of the emerging research area that

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<sup>1</sup>For a comprehensive bibliography of the rapidly growing body of literature on the discursive paradox, see List (2005a).

<sup>2</sup>Also known as the paradox of voting, it is named after the Marquis de Condorcet, who in 1770 proposed a method for the aggregation of preferences. It shows that individual transitive preferences can lead to an intransitive social outcome.

studies the problems of combining individual judgments on logically interconnected propositions into a social judgment on the same propositions. On the pages of this journal Alvin Goldman (2004a) and Christian List (2005b) have developed a judgment aggregation framework for the analysis of institutions in social epistemology. In this paper I will claim that judgment aggregation should not be limited to the investigation of aggregation paradoxes on logically interconnected propositions, but it should also include compound decision problems like the Ostrogorski paradox. In fact, despite the differences, I will show that the latter and the discursive paradox have a similar structure.

In a previous work (Pigozzi 2005) I have illustrated how the inconsistent result of the discursive dilemma can be avoided when we apply an operator defined in artificial intelligence for the merging of belief bases. On the basis of the similarities I will here apply the same merging operator to the Ostrogorski paradox, showing that this inconsistent outcome can also be prevented. The discussion will be informal and conceptual rather than technical.<sup>3</sup> Finally, the application of the same aggregation procedure to these paradoxes will help to clarify the strength and weakness of the aggregation method itself.

The paper is structured as follows: in Section 2 I introduce the discursive paradox and show how the inconsistent results are avoided when we apply one specific merging operator to the discursive dilemma. In Section 3 I explain in what way the Ostrogorski paradox is structurally similar to the discursive dilemma, and how we can prevent counterintuitive outcomes by applying a merging operator to this problem as well. In Section 4 I outline an alternative approach to deal with aggregation paradoxes like the two investigated in the present paper.

## 2 The discursive dilemma

Suppose that a hiring committee in a higher education institution appoints a candidate (proposition  $S$ ) if and only if either the candidate is good at teaching ( $P$ ) and at fund raising ( $Q$ ) or she is good at research ( $R$ ). This complex decision rule can be formally expressed as  $((P \wedge Q) \vee R) \leftrightarrow S$ . The logical connectives among the propositions make the judgments on  $P, Q, R$  (also called the premises) and  $S$  (the conclusion) interdependent. For example, if a member of the committee is in favour of hiring a candidate despite

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<sup>3</sup>The formal definition of the merging operator can be found in Konieczny (1999) and in Konieczny and Pino-Pérez (1998, 2002). The reader should refer to Pigozzi (2005) for the formal application of belief merging to the discursive paradox.

the fact that she is bad at teaching, fund raising and research, that member would be irrational, since her judgment set is inconsistent. A judgment set is (logically) consistent when the judgments on the propositions are consistent according to classical propositional logic.

Assume now that each member of the hiring committee makes a consistent judgment on the propositions  $P$ ,  $Q$ ,  $R$  and  $S$ , and that the individual judgments are combined into a collective one via propositionwise majority voting, viz. the collective judgment on each proposition is the majority judgment on that proposition. A discursive paradox can emerge when the combination of consistent individual judgments is an inconsistent collective outcome. An example of a discursive paradox with a five-member  $(K_1, K_2, \dots, K_5)$  hiring committee is shown in the table below.

$P$ : the candidate is good at teaching  
 $Q$ : the candidate is good at fund raising  
 $R$ : the candidate is good at research  
 $S$ : the candidate is appointed  
 $((P \wedge Q) \vee R) \leftrightarrow S$

	$P$	$Q$	$R$	$S$
$K_1$	0	1	0	0
$K_2$	0	1	0	0
$K_3$	1	0	0	0
$K_4$	1	1	1	1
$K_5$	1	1	1	1
$m$	1	1	0	0

A majority ( $m$ ) in the committee believes both  $P$  and  $Q$  to be true and  $R$  and  $S$  to be false, which is an inconsistent collective set of judgments, since it violates the decision rule  $((P \wedge Q) \vee R) \leftrightarrow S$ . This is an example of the discursive dilemma. Let us now turn to an alternative approach to the aggregation of judgment that avoids the inconsistent outcomes.

## 2.1 Belief merging applied to the discursive dilemma

In order to escape the paradox, two procedures have been suggested. One procedure is to let each member publicly vote on each premise and proceed to appoint the candidate only if a majority of the members believes that the candidate is good at teaching and at fund raising or if a majority believes that the candidate is good at research (this is called the premise-based procedure).

The second procedure is that each member decides about  $P$ ,  $Q$  and  $R$  and then publicly casts her vote on the conclusion  $S$  only if she believes that the candidate is good at teaching and at fund raising or she believes that the candidate is good at research (this is called the conclusion-based procedure). If the latter procedure is followed, the candidate will be appointed if and only if a majority of the committee voted for  $S$ .

Both standard procedures proposed to generate consistent judgments may appear unsatisfactory: conclusion-based voting generates incomplete judgments (by being silent on the premises), and premise-based voting prioritizes some propositions (the premises) over others (the conclusion). A natural procedure to generate consistent and complete judgment sets without any prioritization is to apply a merging operator, initially introduced in artificial intelligence to merge several finite sets (bases) of propositions. The justification for this move is that belief merging and group decision-making share a similar difficulty, viz. the definition of procedures that produce collective belief from individual belief bases, and operators that produce a collective decision from individual decisions.

One of the major problems in contemporary artificial intelligence is the aggregation of finite sets of propositions. In many applications it is necessary to merge beliefs from different and potentially conflicting sources in order to obtain a collective belief representation (an example is the definition of an expert system from a group of human experts). The outcome of a merging process is a consistent set that integrates (parts of) the initial bases. When the individual bases are mutually consistent, the collective outcome can easily be constructed: it is the union of the individual belief bases. Things are more challenging when the individual belief bases are in conflict with each other.

Formally, a belief base  $K_i$  is a finite set of propositional formulas representing the explicit beliefs of the individual  $i$ . For example, if the individual  $i$  believes that the propositions  $P$  and  $Q$  are true, the corresponding belief base is  $K_i = \{P \wedge Q\}$ . A merging operator combines various and possibly conflicting belief bases. An example of three individuals with conflicting beliefs about the propositions  $P$  and  $Q$  is:  $K_1 = \{P \wedge \neg Q\}$ ,  $K_2 = \{\neg P \wedge Q\}$  and  $K_3 = \{\neg P \wedge \neg Q\}$ .

Beliefs are often interconnected. It is easy to imagine examples where the propositions  $P$  and  $Q$  above cannot be both true. It can be something on which all the individuals of the group agree, or they know it is true (e.g., the same person cannot be in two different places at the same time). It can also express a condition accepted by all the members (e.g. we cannot go both to the cinema and to the theatre tonight). In both cases, we require that also

the collective outcome satisfies this condition. These kinds of conditions are called *integrity constraints (IC)*.

Let us now go back to the five-member hiring committee example. There, the only *IC* is the rule that states that the candidate will be appointed only if she is good at teaching and at fund raising or she is good at research, i.e.  $((P \wedge Q) \vee R) \leftrightarrow S$ . We have seen that the discursive paradox arises when the aggregation of consistent individual judgments sets (that is, judgments that satisfy the *IC*) is an inconsistent social judgment set (that is, a collective judgment that fails to satisfy the *IC*). The merging operator avoids inconsistent results by imposing the *IC* on the collective outcomes.

In order to distinguish the consistent outcomes from the inconsistent ones, we need to introduce two concepts from elementary logic: the notions of *interpretation* and *model*. An interpretation is an assignment of a truth value (0/false or 1/true) to each propositional variable, and it is represented as the list of these binary evaluations: for instance, given four propositional variables  $P$ ,  $Q$ ,  $R$  and  $S$ , the vector  $(1,0,0,0)$  stands for the interpretation in which  $P$  is true and  $Q$ ,  $R$  and  $S$  are each false. An interpretation determines a unique truth value for any (possibly compound) proposition: for instance, under the interpretation  $(1,1,0,1)$ ,  $Q \wedge R$  is false and  $((P \wedge Q) \vee R) \leftrightarrow S$  is true. Finally, an interpretation is a model of a propositional formula if and only if it makes the formula true in the usual classical truth functional way. For example, the interpretation  $(1,1,0,1)$  is a model of  $((P \wedge Q) \vee R) \leftrightarrow S$ , but it is not a model of  $Q \wedge R$ . Conversely, the set of models of a formula is the set of *all* the interpretations that make that formula true. For instance, the set of models of  $P \vee Q$  is  $\{(1, 1), (1, 0), (0, 1)\}$ .

In order to define a merging operator we need to specify how the belief bases are combined into a collective one. We can anticipate that a collective outcome under the new aggregation procedure is a model of the *IC* (i.e. a consistent truth value assignment to the premises *and* to the conclusion in discursive paradox terms).

Let  $E = (K_1, K_2, \dots, K_n)$  be a finite collection of belief bases  $K_i$  each representing an individual judgment set.<sup>4</sup> The aggregation rule  $\Delta$  is a function that assigns a collective belief base (which corresponds to a consistent collective judgment set) to the *IC* and to  $E$ . The result is denoted by  $\Delta_{IC}^E$ .

Several types of merging operators have already been proposed in the literature. The one used here is intended to reflect the view of the majority, by maximizing the level of total agreement among the individuals. This is

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<sup>4</sup>The order of the  $K_i$ s is not relevant.

also the operator that behaves most similarly to the propositionwise majority voting adopted in the discursive dilemma (and, as we will see, in the Ostrogorski paradox).<sup>5</sup>

Intuitively, the models of the merged bases are models of  $IC$ , which are preferred according to some distance measure. The so-defined merging operator chooses the outcome so as to minimize its sum of distances to the individual bases. The distance measure Konieczny and Pino-Pérez use is the widely known Dalal's distance (Dalal 1988a, 1988b) between two interpretations. According to this measure, the distance between two interpretations is equal to the number of the propositional variables in which two interpretations differ. For example,  $w = (1, 1, 0, 1)$  and  $w' = (1, 0, 1, 1)$  are two models of  $((P \wedge Q) \vee R) \leftrightarrow S$ , and the distance between  $w$  and  $w'$  is 2 (as they assign a different truth value to  $Q$  and to  $R$ , while they agree on  $P$  and  $S$  being true).

We are now ready to apply the belief merging to the five-member committee problem.  $E$  is  $(K_1, K_2, K_3, K_4, K_5)$ , where each  $K_i$  is the belief base of a member of the hiring committee, and  $IC = \{((P \wedge Q) \vee R) \leftrightarrow S\}$ . The merging procedure takes each individual judgment set as a belief base. Each individual makes a judgment over the atomic propositions  $P$ ,  $Q$ ,  $R$  and  $S$  that satisfies the  $IC$ . We can therefore write:

$$\begin{aligned} K_1 = K_2 &= \{\neg P, Q, \neg R, \neg S\} \\ K_3 &= \{P, \neg Q, \neg R, \neg S\} \\ K_4 = K_5 &= \{P, Q, R, S\} \end{aligned}$$

The model of  $K_1$  and  $K_2$  is  $(0, 1, 0, 0)$ , the model of  $K_3$  is  $(1, 0, 0, 0)$  and the model of  $K_4$  and  $K_5$  is  $(1, 1, 1, 1)$ .

The table below shows the result of the  $IC$  majority merging operator on  $E = (K_1, K_2, K_3, K_4, K_5)$ . In the first column are all the models of  $IC$ . The numbers in the columns  $K_1$ ,  $K_2$ ,  $K_3$ ,  $K_4$  and  $K_5$  are the Dalal's distances of each  $K_i$  from the respective model. Finally, the last column contains the sum of the numbers expressing the distance between the corresponding interpretation on the four propositions and each belief base  $K_i$  in  $E$ .

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<sup>5</sup>Model-based merging operators like the one used here have been discussed in Revesz (1997), Konieczny and Pino-Pérez (1998), Lin and Mendelzon (1996, 1999), Liberatore and Schaerf (1998).

	$K_1$	$K_2$	$K_3$	$K_4$	$K_5$	$\Delta_{IC}^E$
(1,1,1,1)	3	3	3	0	0	9
(1,1,0,1)	2	2	2	1	1	8
(1,0,1,1)	4	4	2	1	1	12
(1,0,0,0)	2	2	0	3	3	10
(0,1,1,1)	2	2	4	1	1	10
(0,1,0,0)	0	0	2	3	3	8
(0,0,1,1)	3	3	3	2	2	13
(0,0,0,0)	1	1	1	4	4	11

Only the models in the first column of the table are the available candidates for the collective judgments. The merging operator selects the models associated with the total minimum distance value (that is, 8). These are (1, 1, 0, 1) and (0, 1, 0, 0) (in the rows with a shaded background), which is tantamount to saying that the collective outcome is a tie. This result can be interpreted by saying that we do not have enough information to select a unique collective judgment. We therefore avoid the inconsistency at the price of indecision.

We now turn to the Ostrogorski paradox and show how this has a similar structure with the discursive paradox, thus justifying the application of the belief merging operator also to this problem.

### 3 The Ostrogorski paradox

Let us consider a two-party system (government and opposition) and three issues (economic, environmental, international). The two parties have opposite views on the issues, and each individual casts a vote (1 or 0) depending on whether she wishes a policy change on that issue (so she agrees with the opposition on that issue), or the government represents her opinion on that matter (no policy change). Suppose there are five voters and that they vote as in the table below:

	Econ	Env	Int	Party
$K_1$	0	1	0	0
$K_2$	0	1	0	0
$K_3$	1	0	0	0
$K_4$	1	1	1	1
$K_5$	1	1	1	1
$m$	1	1	0	0

The table has the same structure as the one used in the five-members hiring committee example of the discursive paradox. The propositions  $P$ ,  $Q$ ,  $R$  and  $S$  are now replaced by the three issues and the party. The aggregation method in the Ostrogorski paradox is different. If in the discursive dilemma the relationships among the propositions  $P$ ,  $Q$ ,  $R$  and  $S$  were of a logical kind, the Ostrogorski paradox is a compound majority decision. Each voter votes for the government (resp. opposition) if she agrees with the government (resp. opposition) on a majority of the issues.

Like the discursive paradox, the Ostrogorski case is puzzling because, despite the individuals being rational, the collective outcome is inconsistent. If each voter votes for the party with which she agrees on a majority of issues, the government wins. However, the opposition represents the views of the majority of the voters on every issue (specifically on the economic and the environmental policy).

The argumentative structure of the Ostrogorski paradox is similar to the premises-conclusion structure of the discursive dilemma. Here an individual votes for the government (opposition) if and only if she agrees with the government (opposition) on at least two of the three issues. As we have seen, in the discursive paradox a member of the committee would appoint a candidate ( $S$ ) if and only if she believes that the candidate is good at teaching ( $P$ ) and at fund raising ( $Q$ ) or the candidate is good at research ( $R$ ). A candidate can be appointed ( $S$  true) for various reasons ( $R$  is believed to be true by at least three members in the committee, or both  $P$  and  $Q$  are believed to be true by a majority in the committee). Similarly, in the Ostrogorski paradox the reasons to vote for the government (opposition) could be one among several judgment sets on the three issues  $(0,0,0)$ ,  $(0,0,1)$ ,  $(0,1,0)$ , etc. (respectively  $(1,1,1)$ ,  $(1,1,0)$ ,  $(1,0,1)$ , etc. for the opposition).

However, there are two differences between the paradoxes. The first is that in the Ostrogorski case the individuals are asked to vote, not to make a judgment in the form of true/false on some propositions. The individuals may cast their votes depending on their desires without committing themselves to a truth-conducive vote.<sup>6</sup> The second difference is that in the Ostrogorski paradox the issues and the parties are not logically connected (as the propositions in the discursive paradox are). It is rather a majority rule on the issues that determines which party an individual should vote for: the individual votes for the party she is in agreement with on the majority of the issues.

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<sup>6</sup>For an epistemic perspective to the discursive paradox see Bovens and Rabinowicz (2004).

The Ostrogorski paradox and the discursive dilemma are two problems arising when individual consistent votes and judgments are combined into a social decision. The interdependence of the propositions in the two paradoxes are not of the same kind. In the Ostrogorski paradox it is a compound majority decision that binds parties and issues, whereas in the discursive dilemma the relationship among propositions is purely logical. The dependence relations are different, however, the reason for the paradoxes arising is that in both cases the majority is applied at different levels, as the figure below illustrates.

	P	Q	R	S
	Econ	Env	Int	Party
$K_1$	0	1	0	0
$K_2$	0	1	0	0
$K_3$	1	0	0	0
$K_4$	1	1	1	1
$K_5$	1	1	1	1
$m$	1	1	0	0

$\xrightarrow{F(m(\bullet), \dots, m(\bullet))=1}$

$\downarrow m(F(\bullet))=0$

Let  $F$  be a variable for the two functions that assign the appropriate value 0 or 1 to the conclusion  $S$  (resp. the party), given the values for the premises (resp. the issues).  $F$  is the compound majority decision rule in the Ostrogorski problem, and the truth value function that associates 1 to  $S$  if and only if  $(P \wedge Q) \vee R$  is true in the discursive paradox. Let  $m$  be the majority rule. The paradoxes arise because the function  $F$  on the premises (issues) that received the greater support (i.e.,  $F(m(\bullet), \dots, m(\bullet))$ ) is not necessarily the same as the majority voting rule applied to the function's value of the conclusion (party) (i.e.,  $m(F(\bullet))$ ).

### 3.1 Belief merging applied to the Ostrogorski paradox

I will now show how the inconsistent result of the Ostrogorski paradox can as well be avoided when we apply the majority merging operator introduced in the previous section.

Alike to the representation of the interpretations of propositional formulas as lists of binary evaluations, individual preferences can be also represented as binary vectors. For example, in the Ostrogorski paradox the individuals  $K_1$  and  $K_2$ , though they agree with the opposition on the environmental issue

(1), do not wish a policy change (0) on the economic and the international issues and, therefore, vote for the government. The corresponding binary vector for  $K_1$  and  $K_2$  is  $(0, 1, 0, 0)$ . The voter  $K_3$  wishes a change only with regard to the economic issue, but is happy with the government on both the environmental and the international policies and, therefore, votes for the government.  $K_3$ 's preferences are represented by  $(1, 0, 0, 0)$ . Finally,  $K_4$  and  $K_5$  vote for a policy change on all the three issues and, consequently, they vote for the opposition. Their preferences can be expressed by  $(1, 1, 1, 1)$ .

The set of the permissible social outcomes, i.e. those outcomes where a party is elected only if it accumulated the majority on the issues, is the following:

$$\{(1, 1, 1, 1), (1, 1, 0, 1), (1, 0, 1, 1), (1, 0, 0, 0), (0, 1, 1, 1), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 0)\}$$

These are in the first column of the table below. When we apply the majority belief merging we obtain the following result.

	$K_1$	$K_2$	$K_3$	$K_4$	$K_5$	$\Delta_{IC}^E$
$(1,1,1,1)$	3	3	3	0	0	9
$(1,1,0,1)$	2	2	2	1	1	8
$(1,0,1,1)$	4	4	2	1	1	12
$(1,0,0,0)$	2	2	0	3	3	10
$(0,1,1,1)$	2	2	4	1	1	10
$(0,1,0,0)$	0	0	2	3	3	8
$(0,0,1,0)$	2	2	2	3	3	12
$(0,0,0,0)$	1	1	1	4	4	11

We can notice that it is the same table as the hiring committee example, with the difference that the binary valuation  $(0, 0, 1, 1)$  is not any longer an outcome allowed by the compound majority rule in the Ostrogorski paradox, and it has been replaced by  $(0, 0, 1, 0)$ . In fact,  $F(0, 0, 1) = 1$  when  $F$  is the function that assigns a truth value to  $S$  given that  $S$  is equivalent to  $(P \wedge Q) \vee R$ . But  $F(0, 0, 1) = 0$  when  $F$  is the majority rule used in the Ostrogorski paradox. Nevertheless the final outcome is not affected by this change, since the two solutions that minimize the distances are again  $(1, 1, 0, 1)$  and  $(0, 1, 0, 0)$ , with minimum distance 8.

## 4 Floating conclusions

The Ostrogorski paradox (resp. the discursive dilemma) treats the issues and the party (resp. premises and conclusions) on which the individuals are requested to vote (resp. make their judgments) in an even handed manner. Hence the difference between those propositions that are ‘premises’ (‘issues’) and those that are ‘conclusions’ (‘party’) vanishes. The same is true for belief merging, the approach I have argued for: the distance between two interpretations is the number of propositions on which the two interpretations differ, regardless whether they assign a different value to a premise (issue) or to a conclusion (party). (It is indeed worth mentioning that - if the inconsistent outcomes were not excluded by *IC* - the distance-based belief merging operator would assign the minimum distance exactly to the same inconsistent result found via propositionwise majority voting.)

However, the two paradoxes considered here have an argumentative structure; the judgments and the votes on some propositions constrain the judgments and the votes on other propositions. I sketch here a possible framework to refer to if we really intend to grant that these propositions are of a different type, and we therefore want to maintain the argumentative structure of the two paradoxes.

The problem of finding a unique reason for a decision likewise arises in the theory of defeasible inheritance nets (Horty *et al.* 1990, Horty 1994). There, Makinson and Schlechta (1991) first studied the *floating conclusion* phenomenon, which became relevant also in argument systems. A conclusion is floating when it can be supported by two different and potentially incompatible arguments. Floating conclusions can be traced in the discursive dilemma and in the Ostrogorski paradox. In the first paradox we had that the conclusion (*S*) to hire a candidate could be - for example - because of *P* and *Q* are believed to be true, or because *R* is believed to be true. Similarly, in the Ostrogorski paradox a party can win when supported by different combinations of issues.

The theory of defeasible inheritance nets deals with the problem that a belief base can be associated with multiple extensions (an extension represents some total set of arguments), and it is not clear which extension one should adopt. The two solutions proposed for this problem are the *credulous* and the *skeptical* approaches. The credulous approach consists in accepting the set of conclusions endorsed by an arbitrary argument extension. The skeptical approach considers the intersection of the extensions (Horty 2002). Moreover, Makinson and Schlechta observed that there are two ways to perform a skeptical solution. We can either intersect the arguments, or we can

intersect the conclusions. These two skeptical approaches can yield different results, since a statement may be supported in each extension, but only by different and possibly conflicting arguments (floating conclusion). In the literature no final agreement has yet been reached about whether we should always endorse a floating conclusion or not.

An example that illustrates that the two skeptical approaches lead to two different conclusions is the famous Horty's inheritance example. Suppose that John wants to buy a very expensive yacht but he does not have all the money. His parents are very sick and about to die within a month. He has a brother and sister, equally reliable. His brother tells him that their mother will leave half a million dollars to him but their father will give that amount to John. On the other hand, his sister tells him that their mother will leave half a million dollars to John and their father will leave that amount to her.

Horty concludes that if John were to intersect the arguments, he would not be justified in concluding that he is about to inherit half a million dollars. But if he were to follow the second skeptical strategy (intersecting the conclusions), he would be justified in drawing the conclusion that he is about to inherit all that money, and so he could place the deposit for the yacht. Indeed, both his brother's and his sister's arguments, though contradicting each other, support the conclusion that John will inherit half a million dollars either from his father or from his mother.

When we count (as we do in the aggregation paradoxes) how many people in a group voted for a certain conclusion or for a certain party, we ignore the reasons supporting that decision. The literature on floating conclusions shows that splitting a set of arguments into reasons and conclusions, and 'aggregating' them separately, can lead to opposing consequences. I believe that the research done on defeasible inheritance nets can shed new light on the area of judgment aggregation. I plan to investigate this relation in a future paper.

## 5 Conclusions

This paper aimed at showing that a voting paradox and a judgment aggregation paradox have a similar structure. The common feature is that in both cases group members have to express a judgment or cast a vote on several interconnected items.

The rules that make the items interdependent are of a different kind. In the discursive paradox it is a logical rule; the propositions are logically

connected and the individual as well as the collective judgment sets are assumed to be logically consistent. The truth values assigned to the premises entail the truth or falsity of the conclusion. In the Ostrogorski paradox the individual and collective choices on the issues determine which party that person and the group support. In both paradoxes the values assigned to some propositions (premises in the case of the discursive paradox, issues in the Ostrogorski paradox) dictate the individual and collective values on other propositions (conclusions and parties, respectively).

The difficulty with the aggregation problems is that the set of propositions on which most group members agree is not guaranteed to be a candidate for the collective decision. The set can fail to satisfy the dependence relations among the items even though each member consistently expressed her judgments or votes.

One way to avoid this undesirable result is to require that the social outcome satisfies the consistency requirements as the individuals bases do. This is achieved by rejecting the inconsistent results from the set of candidates of the aggregation procedure. I have elsewhere claimed (Pigozzi 2005) that judgment aggregation and belief merging are related in an interesting way, and that more exchange between these two areas of research is definitely desirable. A majoritarian *IC* merging operator for belief bases has been applied to the discursive paradox, and this proved to prevent the paradox. The value of the merging procedure rests upon the exclusion of inconsistent sets of judgments from the set of the candidates apt to become collective judgments, and in the definition of a preference order (induced by a distance measure) on the remaining candidates.

Relying on the similarities between the discursive dilemma and the Ostrogorski paradox, the same merging operator has been applied to the latter dilemma in the present paper. The application of the merging procedure is not the only consequence that can be driven from the observation of the similarities between the two paradoxes. Though the Ostrogorski paradox cannot be strictly defined as a judgment aggregation paradox, the recognition that the cause of the puzzling results resides in which order the majority rule and the specific aggregation function is applied, should make them be recognized as part of a larger class of aggregation problems. Therefore, judgment aggregation should not be restricted to a particular structure of judgment aggregation problems, where the dependence relations are of a logical type, but be broadened to include other types of dependencies.

In the last section I sketched a possible alternative approach to the aggregation problems. This makes use of some of the discussions on floating

conclusions in the area of defeasible inheritance nets. This framework can be interesting for those who believe that an aggregation that handles the items to be combined as of the same kind can be unjustified when dealing with problems as those that we have addressed here.

The discursive dilemma and the Ostrogorski paradox occur because the aggregations on the premises (issues) and on the conclusion (party) go in two divergent directions. On the one hand, the premises that receive the highest degree of support cannot consistently provide reasons for the most popular conclusion (party). On the other hand, the socially selected conclusion cannot be inferred from the premises that received the majority of the votes.

The need for a unique set of reasons for a certain decision is shared by the multiple extensions problem in defeasible inheritance nets. There, Makinson and Schlechta have baptized ‘floating conclusions’ the phenomenon according to which a conclusion is supported by some arguments contained in every extension but there exists no argument in all the extensions that supports that conclusion. I believe that the interplay between aggregation problems and defeasible inheritance nets is worth being investigated. I leave this to future research plans.

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