On the issue of non-interference in the aspic-light formalism

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Abstract

In this paper we consider the argumentation formalism of ASPIC system. We illustrate some problems in argumentation framework and five rationality postulates that can be seen as principles of an argumentation system. Then we provide a solution to avoid these problems and to satisfy those rationality postulates.

1 Introduction

The field of formal argumentation can be traced back to the work of [9, 11], [18, 19], and [15]. The idea is that (nonmonotonic) reasoning can be performed by constructing and evaluating arguments, which are composed of a number of reasons for the validity of a claim. Arguments distinguish themselves from proofs by the fact that they are defeasible, that is, the validity of their conclusions can be disputed by other arguments. Whether a claim can be accepted therefore depends not only on the existence of an argument that supports this claim, but also on the existence of possible counter arguments, that can then themselves be attacked by counter arguments, etc.

Nowadays, much research on the topic of argumentation is based on the abstract argumentation theory of [6]. The central concept in this work is that of an *argumentation framework*, which is essentially a directed graph in which the arguments are represented as nodes and the attack relation is represented by the arrows. Given such a graph, one can then examine the question on which set(s) of arguments can be accepted: answering this question corresponds to defining an *argumentation semantics*. Various proposals have been formulated in this respect, and in the current paper we will describe some of the mainstream approaches. It is, however, important to keep in mind that the issue of argumentation semantics is only one specific aspect (although an important one) in the overall theory of formal argumentation. For instance, if one wants to use argumentation theory for the purpose of (nonmonotonic) entailment, one can distinguish three steps (see Figure 1). First of all, one would use an underlying knowledge base to generate a set of arguments and determine in which ways these arguments attack each other (step 1). The result is then an argumentation framework, to be represented as a directed graph in which the internal structure of the arguments, as well as the nature of the attack relation has been abstracted away. Based on this argumentation framework, the next step is to

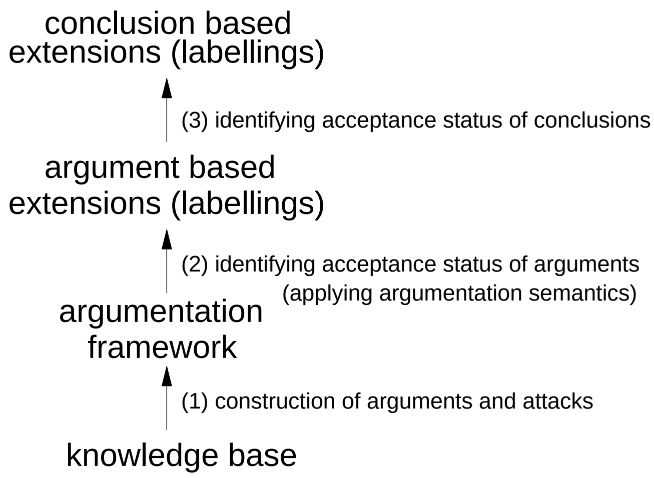

Figure 1: Argumentation for inference

determine the sets of arguments that can be accepted, using a pre-defined criterion corresponding to an argumentation semantics (step 2). After the set(s) of accepted arguments have been identified, one then has to identify the set(s) of accepted conclusions (step 3), for which there exist various approaches.

As illustrated in Figure 1, the argumentation approach provides a graph based way of performing non-monotonic reasoning. An interesting phenomenon is that the non-monotonicity is isolated purely in step 2 of the process. Step 1 is monotonic (having additional information in the knowledge base yields an argumentation framework with zero or more additional vertices and edges), just like step 3 is monotonic (having additional arguments in an argument-based extension yields an associated conclusion-based extension with zero or more additional conclusions). Step 2, however, is non-monotonic because adding new arguments and attacks can change the status of arguments that were already present in the argumentation framework when it comes to determining the argument-based extensions. That is, when adding new arguments and attacks it is by no means guaranteed that the resulting argument-based extensions will be supersets of the previous argument-based extensions. Apart from isolating non-monotonicity in step 2, the argumentation approach to NMR also offers the advantage of different levels of abstraction. The field of abstract argumentation, for instance, only studies step 2 of the overall argumentation process and has now become one of the most popular topics in argumentation research.

Despite its advantages, the argumentation approach to non-monotonic reasoning also has important difficulties that are often overlooked by those studying purely abstract argumentation. The point is that in step 1 of the overall argumentation process, one constructs arguments that have a logical content. Yet, in step 2, one selects the sets of accepted arguments (argument-based extensions) purely based on some topological principle of the resulting graph, without looking what is actually inside of the arguments. The abstract level (step 2) is essentially about how to apply a semantics "blindly", without looking at the logical content of the arguments. But if one cannot see what is inside of the arguments, then how can one make sure that the selected set of arguments makes sense from a logical perspective? For instance, how can one be sure that the conclusions yielded by these sets of arguments (step 3) will be consistent?[1] Or, alternatively, how does one know that these conclusions will actually be closed under logical entailment?

Issues like that of consistency and closure of argumentation-based entailment cannot purely be handled purely at the level of any of the individual three steps in the overall argumentation process. Instead, they require a carefully selected *combination* of how to carry out *each* of these individual steps. For instance, Caminada and Amgoud [5, 1] point out that when applying the argumentation process to a knowledge base consisting of strict and defeasible rules, one can obtain closure and consistency of the resulting conclusions by applying transposition and restricted rebut when constructing the argumentation framework (step 1), in combination with any admissibility-based argumentation semantics (step 2). Under these conditions, the conclusions associated with the argument-based extensions (step 3) will be consistent and closed under the strict rules [1].

Caminada and Amgoud introduce thee postulates that they aim to satisfy for argument-based entailment: *direct consistency*, *indirect consistency* and *closure*. The current paper extends this line of research by providing two additional postulates: *crash resistance* and *non-interference*. It is explained why these postulates matter, and how they are in fact violated by several well-known formalisms for argument-based entailment (including Pollock's OSCAR system [11] and the ASPIC+ system [13]. Furthermore, we provide a general way of satisfying the postulates of Caminada and Amgoud [5, 1] as well as the additional postulates introduced in this paper, in the context of argumentation formalisms that apply defeasible argument schemes in combination with classical logic (like [9, 11] and [13].

The remaining part of this paper is structured as follows. In section 2, we first state the existing postulates of Caminada and Amgoud [5, 1], as well as the additional postulates of crash-resistance and non-interference. We explain their importance, and examine how they are nevertheless violated by formalisms like [11, 13, 14]. In Section 3, we then provide a general solution for satisfying all five postulates (direct consistency, indirect consistency, closure, crash-resistance and non-interference). In Section 4, we round off with a discussion of the obtained results, and compare them with related research.

2 Preliminaries

In this paper we treat the same argumentation formalism called the ASPIC system as it was published in [1]. In this section we will briefly restate some preliminaries regarding Dung's abstract argumentation semantics. Then we restate the ASPIC system and postulates that are treated in [1].

[1]To make an analogy, consider the (fictitious) case of uncle Bob who lives in a retirement home. Every day, he has to take a number of medicines, which come in small bottles that a nurse puts on the table for him. However, some combinations of medicines are poisonous when taken at the same time. Having lost his reading glasses, uncle Bob is unable to read the labels, to determine the actual contents of the medicines. Instead, he chooses which medicines to take purely on how the bottles have been arranged on his table, hoping that the nurse somehow knows his selection criterion and has arranged the bottles accordingly.

2.1 Argument Semantics and Argument Labellings

In this section, we briefly restate some preliminaries regarding Dung's abstract argumentation semantics. For simplicity, we only consider finite argumentation frameworks.

Definition 1. *An* argumentation framework *is a pair* (Ar, att) *where* Ar *is a finite set of arguments and* $att \subseteq Ar \times Ar$.

An argumentation framework can be represented as a directed graph in which the arguments are represented as nodes and the defeat relation is represented as arrows. representation.

Definition 2 (defense / conflict-free)**.**
Let (Ar, def) *be an argumentation framework,* $A \in Ar$ *and* $\mathcal{A}rgs \subseteq Ar$.
We define A^+ *as* $\{B \mid A\ def\ B\}$ *and* $\mathcal{A}rgs^+$ *as* $\{B \mid A\ def\ B$ *for some* $A \in \mathcal{A}rgs\}$.
We define A^- *as* $\{B \mid B\ def\ A\}$ *and* $\mathcal{A}rgs^-$ *as* $\{B \mid B\ def\ A$ *for some* $A \in \mathcal{A}rgs\}$.
$\mathcal{A}rgs$ *is* conflict-free *iff* $\mathcal{A}rgs \cap \mathcal{A}rgs^+ = \emptyset$.
$\mathcal{A}rgs$ defends *an argument* A *iff* $A^- \subseteq \mathcal{A}rgs^+$.
Let $F : 2^{Ar} \rightarrow 2^{Ar}$ *be the function defined as:* $F(\mathcal{A}rgs) = \{A \mid A$ *is defended by* $\mathcal{A}rgs\}$.

Definition 3 (acceptability semantics)**.** *Let* (Ar, def) *be an argumentation framework. A conflict-free set* $\mathcal{A}rgs \subseteq Ar$ *is called*

- *an* admissible set *iff* $\mathcal{A}rgs \subseteq F(\mathcal{A}rgs)$.
- *a* complete extension *iff* $\mathcal{A}rgs = F(\mathcal{A}rgs)$.
- *a* grounded extension *iff* $\mathcal{A}rgs$ *is a minimal complete set.*
- *a* preferred extension *iff* $\mathcal{A}rgs$ *is a maximal complete set.*
- *a* stable extension *iff* $\mathcal{A}rgs$ *is a complete set that defeats every argument in* $Ar \backslash \mathcal{A}rgs$.

The concept of complete semantics was originally stated in terms of sets of arguments. It is equally well possible, however, to express this concept in terms of *argument labellings.* The approach of (argument) labellings has been used by Pollock [11] and by Jakobovits and Vermeir [7], and has recently been extended by Caminada [4], Vreeswijk [20] and Verheij [16]. The idea of a labelling is to associate with each argument exactly one label, which can either be `in`, `out` or `undec`. The label `in` indicates that the argument is explicitly accepted, the label `out` indicates that the argument is explicitly rejected, and the label `undec` indicates that the status of the argument is undecided, meaning that one abstains from an explicit judgment whether the argument is `in` or `out`. For complete, grounded, preferred, stable, semi-stable and ideal semantics, it holds that labellings and extensions stand in a one to one relationship with each other [**?**, **?**]. In essence, labellings and extensions are different ways to describe the same concept.

2.2 Constructed Arguments

The ASPIC system instantiates Dung's abstract argumentation framework by given arguments structures consisting of strict rules and defeasible rules and is built around a underlying logical language $\mathcal{L}$.

Definition 4. *Let $\mathcal{L}$ be a logical language and $-$ be a function from $\mathcal{L}$ to $2^{\mathcal{L}}$ such that $-\psi = \phi$ iff $\psi = \neg\phi$ and $-\psi = \neg\phi$ iff $\psi = \phi$.*

Definition 5. *Let $\mathcal{P} \subseteq \mathcal{L}$. $\mathcal{P}$ is consistent iff $\nexists\ \psi, \varphi \in \mathcal{P}$ such that $\psi = -\varphi$.*

Arguments consist of strict or defeasible rules [8, 12, 17].

Definition 6. *Let $\varphi_1, \ldots, \varphi_n, \varphi \in \mathcal{L}$ $(n \geq 0)$.*

- *A* strict rule *is an expression of the form $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$, indicating that if $\varphi_1, \ldots, \varphi_n$ hold, then* without exception *it holds that φ.*
- *A* defeasible rule *is an expression of the form $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$, indicating that if $\varphi_1, \ldots, \varphi_n$ hold, then* usually *it holds that φ.*

Definition 7. *Let $\mathcal{P} \subseteq \mathcal{L}$. The closure of $\mathcal{P}$ under the set $\mathcal{S}$ of strict rules, denoted $\mathcal{C}l_{\mathcal{S}}(\mathcal{P})$, is the smallest set such that:*

- *$\mathcal{P} \subseteq \mathcal{C}l_{\mathcal{S}}(\mathcal{P})$.*
- *if $\phi_1, \ldots, \phi_n \rightarrow \psi \in \mathcal{S}$ and $\phi_1, \ldots, \phi_n \in \mathcal{C}l_{\mathcal{S}}(\mathcal{P})$ then $\psi \in \mathcal{C}l_{\mathcal{S}}(\mathcal{P})$.*

Definition 8. *A* inference base *$\mathcal{B}$ is a pair $(\mathcal{P}, \mathcal{D})$ where $\mathcal{P} \subseteq \mathcal{L}$ and $\mathcal{D}$ is a set of defeasible rules.*

Definition 9. *A defeasible theory $\mathcal{T}$ based on a inference base $\mathcal{B} = (\mathcal{P}, \mathcal{D})$ is a pair $(\mathcal{S}, \mathcal{D})$ such that $\mathcal{S} = \{\rightarrow \varphi \mid \varphi \in \mathcal{P}\} \cup \{\varphi_1, \ldots, \varphi_n \rightarrow \varphi \mid \varphi_1, \ldots, \varphi_n \vdash \psi$ and $\varphi_1, \ldots, \varphi_n, \psi \in \mathcal{L}\}$.*

An argument can be built from a defeasible theory. The conclusion of an argument is returned by a function Conc which is the head of the root rule of the argument. Sub returns all sub-arguments of the argument and functions StrictRules and DefRules return all the strict rules and the defeasible rules respectively.

Definition 10. *Let $\mathcal{T} = (\mathcal{S}, \mathcal{D})$ be a defeasible theory. An argument A constructed from $\mathcal{T}$ is:*

- *$A_1, \ldots, A_n \rightarrow \psi$ $(n \geq 0)$ if $A_1, \ldots, A_n$ are arguments such that there exists a strict rule $r \in \mathcal{S}$ and $r = \text{Conc}(A_1), \ldots, \text{Conc}(A_n) \rightarrow \psi$.*

  $\text{Conc}(A) = \psi$,

  $\text{TopRule}(A) = r$,

  $\text{Sub}(A) = \text{Sub}(A_1) \cup \ldots \cup \text{Sub}(A_n) \cup \{A\}$,

  $\text{StrictRules}(A) = \text{StrictRules}(A_1) \cup \ldots \cup \text{StrictRules}(A_n) \cup \{\text{Conc}(A_1), \ldots, \text{Conc}(A_n) \rightarrow \psi\}$,

  $\text{DefRules}(A) = \text{DefRules}(A_1) \cup \ldots \cup \text{DefRules}(A_n)$.

- $A_1, \ldots, A_n \Rightarrow \psi$ $(n \geq 0)$ *if* $A_1, \ldots, A_n$ *are arguments such that there exists a defeasible rule* $r \in \mathcal{D}$ *and* $r = \mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \Rightarrow \psi$.

  $\mathrm{Conc}(A) = \psi$,

  $\mathrm{TopRule}(A) = r$,

  $\mathrm{Sub}(A) = \mathrm{Sub}(A_1) \cup \ldots \cup \mathrm{Sub}(A_n) \cup \{A\}$,

  $\mathrm{StrictRules}(A) = \mathrm{StrictRules}(A_1) \cup \ldots \cup \mathrm{StrictRules}(A_n)$,

  $\mathrm{DefRules}(A) = \mathrm{DefRules}(A_1) \cup \ldots \cup \mathrm{DefRules}(A_n) \cup \{\mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \Rightarrow \psi\}$.

Let $\mathcal{A}rgs$ *be the set of all arguments that can be built from* $\mathcal{T}$ *and let* $A, A' \in \mathcal{A}rgs$.

- A' *is a* subargument *of* A *iff* $A' \in \mathrm{Sub}(A)$.

- A' *is a* direct subargument *of* A *iff* $A' \in \mathrm{Sub}(A)$, $A' \neq A$, $\nexists A'' \in \mathcal{A}rgs$ *such that* $A'' \in \mathrm{Sub}(A)$ *and* $A' \in \mathrm{Sub}(A'')$, $A \neq A''$ *and* $A' \neq A''$.

An argument is strict if it is constructed only by strict rules, otherwise it is defeasible.

Definition 11. *Let* A *be an argument.* A *is strict iff* $\mathrm{DefRules} = \emptyset$, *otherwise* A *is defeasible.*

The consistency of a set of strict rules is a condition that argumentation frameworks need to fulfill in order to satisfy many postulates.

Definition 12. *Let* $\mathcal{T} = (\mathcal{S}, \mathcal{D})$ *be a defeasible theory and let* $\mathcal{A}rgs$ *be the set of arguments that constructed from* $\mathcal{T}$. $\mathcal{S}$ *is said to be consistent iff* $\nexists A, B \in \mathcal{A}rgs$ *such that* A *and* B *are strict arguments and* $\mathrm{Conc}(A) = -\mathrm{Conc}(B)$.

We use restricted rebutting in [1] for the notion of rebutting which defines that an argument can only be rebutted on the consequent of one of its defeasible rules.

Definition 13. *Let* A *and* B *be arguments.* A *rebuts* B *on* B' *iff* $\mathrm{Conc}(A) = \phi$ *and* $B' \in \mathrm{Sub}(B)$ *such that* B' *is of the form* $B''_1, \ldots, B''_n \Rightarrow -\phi$

Definition 14. *Let* A *and* B *be arguments.* A *undercuts* B *on* B' *iff* $\exists B' \in \mathrm{Sub}(B)$ *such that* B' *is of the form* $B''_1, \ldots, B''_n \Rightarrow \psi$ *and* $\mathrm{Conc}(A) = \neg\lceil B''_1, \ldots, B''_n \Rightarrow \psi \rceil$.

Restricted defeating [1] follows the notion of restricted rebutting.

Definition 15. *Let* A *and* B *be arguments.* A *defeats* B *iff* A *rebuts* B *or* A *undercuts* B.

The following Proposition [1] shows that if an argument is in a given extension, then all its sub-arguments are also in that extension.

Proposition 1. *Let* (Ar, def) *be an argumentation framework, and* $\{E_1, \ldots, E_n\}$ $(n \geq 1)$ *be its set of extensions under one of Dung's standard semantics.* $\forall E_i \in \{E_1, \ldots, E_n\}$, $\forall A \in E_i$, $\mathrm{Sub}(A) \subseteq E_i$.

Proof. Please refer to [1]. □

The justified conclusions [1] are conclusions that are supported by at least one argument in each extension.

Definition 16. *Let* (Ar, def) *be an argumentation framework, and* $\{E_1, \ldots, E_n\}$ $(n \geq 1)$ *be its set of extensions under one of Dung's standard semantics.*

- $\text{Concs}(E_i) = \{\text{Conc}(A) \mid A \in E_i\}$ $(1 \leq i \leq n)$.
- $\text{Output} = \bigcap_{i=1,\ldots,n} \text{Concs}(E_i)$.

2.3 Problems and Postulates

In this section we present five postulates based on a definition of logic formalism. First we describe closure, direct consistency and in direct consistency which are introduced in [1]. Then we introduce the definitions of non-interference and crash resistance [3].

Definition 17. *A* logical formalism *is a triple* $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ *where* $\mathcal{A}toms$ *is a countably (finite or infinite) set of atoms,* $\mathcal{F}ormulas$ *is the set of all well-formed formulas that can be constructed using* $\mathcal{A}toms$*, and* $Cn : 2^{\mathcal{F}ormulas} \to 2^{2^{\mathcal{F}ormulas}}$ *is the consequence function.*

In the rest of the paper we use Atoms($\mathcal{F}$) for the atoms that occur in a set of formulas $\mathcal{F}$. For instance: atoms($\{a \to b; b \to c\}$) = $\{a, b, c\}$. Furthermore, if At is a set of atoms and $\mathcal{F}$ is a set of formulas, then we write $\mathcal{F}_{|At}$ for formulas in $\mathcal{F}$ that contain only atoms from At. For instance: $\{a \to b; b \to c\}_{|\{a,b\}} = \{a \to b\}$. We say that two sets of formulas $\mathcal{F}_1$ and $\mathcal{F}_2$ are *syntactically disjoint* iff $\text{atoms}(\mathcal{F}_1) \cap \text{atoms}(\mathcal{F}_2) = \emptyset$.

Postulate 1. *We say that a logical formalism* $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ *satisfies* closure *iff for every* $\mathcal{F} \subseteq \mathcal{F}ormulas$ *and let* $Cn(\mathcal{F}) = \{C_1, \ldots, C_n\}$ *it holds that*

(1) $C_i = \mathcal{C}l_{\mathcal{S}}(C_i)$ *for each* $1 \leq i \leq n$.

(2) $\bigcap_{i=1}^{n} C_i = \mathcal{C}l_{\mathcal{S}}(\bigcap_{i=1}^{n} C_i)$.

The idea of closure is that the conclusions of an argumentation framework should be complete. If there exists a strict rule $a \to b$ and a is justified then b should be justified too.

An argumentation framework satisfies closure if its set of justified conclusions, as well as the set of conclusions supported by each extension are closed.

Definition 18. *Let* $\mathcal{T}$ *be a defeasible theory,* (Ar, def) *be an argumentation framework built from* $\mathcal{T}$*.* Output *is its set of justified conclusions, and* $E_1, \ldots, E_n$ *its extensions under a given semantics.* (Ar, def) *satisfies closure iff:*

(1) $\text{Concs}(E_i) = \mathcal{C}l_{\mathcal{S}}(\text{Concs}(E_i))$ *for each* $1 \leq i \leq n$.

(2) $\text{Output} = \mathcal{C}l_{\mathcal{S}}(\text{Output})$.

The following Proposition shows that if the different sets of conclusions of the extensions are closed, then the set Output is also closed.

Proposition 2. *Let $\mathcal{T}$ be a defeasible theory, (Ar, def) be an argumentation framework built from $\mathcal{T}$. Let $E_1, \ldots, E_n$ its extensions under a given semantics and* Output *is its set of justified conclusions. If* $\text{Concs}(E_i) = \mathcal{C}l_{\mathcal{S}}(\text{Concs}(E_i))$ *for each* $1 \leq i \leq n$ *then* $\text{Output} = \mathcal{C}l_{\mathcal{S}}(\text{Output})$.

Proof. Please refer to [1]. □

Postulate 2. *We say that a logical formalism $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ satisfies* direct consistency *iff for every $\mathcal{F} \subseteq \mathcal{F}ormulas$ and let $Cn(\mathcal{F}) = \{C_1, \ldots, C_n\}$ it holds that*

(1) C_i is consistent for each $1 \leq i \leq n$.

(2) $\bigcap_{i=1}^{n} C_i$ is consistent.

An argumentation framework satisfies direct consistency if its set of justified conclusions is consistent and each set of conclusions corresponding to its extension is consistent.

Definition 19. *Let $\mathcal{T}$ be a defeasible theory, (Ar, def) be an argumentation framework built from $\mathcal{T}$.* Output *is its set of justified conclusions, and $E_1, \ldots, E_n$ its extensions under a given semantics. (Ar, def) satisfies direct consistency iff:*

(1) $\text{Concs}(E_i)$ *is consistent for each* $1 \leq i \leq n$.

(2) Output *is consistent.*

Postulate 3. *We say that a logical formalism $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ satisfies* indirect consistency *iff for every $\mathcal{F} \subseteq \mathcal{F}ormulas$ and let $Cn(\mathcal{F}) = \{C_1, \ldots, C_n\}$ it holds that*

(1) $\mathcal{C}l_{\mathcal{S}}(C_i)$ is consistent for each $1 \leq i \leq n$.

(2) $\mathcal{C}l_{\mathcal{S}}(\bigcap_{i=1}^{n} C_i)$ is consistent.

If the closure of the set of justified conclusions is consistent and the closure of conclusions of each extension is consistent, then the argumentation framework satisfies indirect consistency.

Definition 20. *Let $\mathcal{T}$ be a defeasible theory, (Ar, def) be an argumentation framework built from $\mathcal{T}$.* Output *is its set of justified conclusions, and $E_1, \ldots, E_n$ its extensions under a given semantics. (Ar, def) satisfies indirect consistency iff:*

(1) $\mathcal{C}l_{\mathcal{S}}(\text{Concs}(E_i))$ *is consistent for each* $1 \leq i \leq n$.

(2) $\mathcal{C}l_{\mathcal{S}}(\text{Output})$ *is consistent.*

Proposition 3 [1] shows that if the closure of conclusions of each extension is consistent, then the closure of the justified conclusions is consistent.

Proposition 3. *Let $\mathcal{T}$ be a defeasible theory, (Ar, def) be an argumentation framework built from $\mathcal{T}$. Let $E_1, \ldots, E_n$ its extensions under a given semantics and* Output *is its set of justified conclusions. If* $\mathcal{C}l_{\mathcal{S}}(\text{Concs}(E_i))$ *is consistent for each* $1 \leq i \leq n$ *then* $\mathcal{C}l_{\mathcal{S}}(\text{Output})$ *is consistent.*

Proof. Please refer to [1]. □

Proposition 4. *Let $AF = (Ar, def)$ be an argumentation framework. If AF satisfies closure and direct consistency, then it satisfies indirect consistency.*

Proof. Please refer to [1]. □

It follows that if indirect consistency is satisfied by an argumentation framework, then the argumentation framework also satisfies direct consistency.

Closure, direct consistency and indirect consistency are three important properties of ASPIC system. They guarantee that the results of extensions of an argumentation framework are sound and complete.

Let's use an example [3] to illustrate the other two postulates non-interference and crash resistance.

Example 1.
$\mathcal{P} = \{Says(J, s)$, *"John says sugar has been added."*
$Says(M, \neg s)$, *"Mary says sugar has not been added."*
$Says(J, unrel(J))$, *"John says John is unreliable."*
$Says(M, unrel(M))$, *"Mary says Mary is unreliable."*
$unrel(J) \supset \neg\lceil Says(J, x) \Rightarrow x\rceil$, *"If John is unreliable then what John says is defeasible."*
$unrel(M) \supset \neg\lceil Says(M, x) \Rightarrow x\rceil$, *"If Mary is unreliable then what Mary says is defeasible."*
$Says(WF, r)\}$, *"The weather forecaster predicts rain today."*
$\mathcal{D} = \{Says(X, y) \Rightarrow y\}$, *"People usually tell the truth."*

Consider the following arguments:
$J_0 : \rightarrow Says(J, s)$
$J_1 : \rightarrow Says(J, unrel(J))$
$J_2 : \rightarrow unrel(J) \supset \neg\lceil Says(J, x) \Rightarrow x\rceil$
$J_3 : Says(J, unrel(J)) \Rightarrow unrel(J)$
$J_4 : J_1, (unrel(J) \supset \neg\lceil Says(J, unrel(J)) \Rightarrow unrel(J)\rceil) \rightarrow$
$\neg\lceil Says(J, unrel(J)) \Rightarrow unrel(J)\rceil$
$J_5 : J_1, (unrel(J) \supset \neg\lceil Says(J, s) \Rightarrow s\rceil) \rightarrow \neg\lceil Says(J, s) \Rightarrow s\rceil$
$J_6 : Says(J, s)$
$J_7 : J_4 \Rightarrow s$
$M_0 : \rightarrow Says(M, \neg s)$
$M_1 : \rightarrow Says(M, unrel(M))$
$M_2 : \rightarrow unrel(M) \supset \neg\lceil Says(M, x) \Rightarrow x\rceil$
$M_3 : Says(M, unrel(M)) \Rightarrow unrel(M)$
$M_4 : M_1, (unrel(M) \supset \neg\lceil Says(M, unrel(M)) \Rightarrow unrel(M)\rceil) \rightarrow$
$\neg\lceil Says(M, unrel(M)) \Rightarrow unrel(M)\rceil$
$M_5 : M_1, (unrel(M) \supset \neg\lceil Says(M, \neg s) \Rightarrow \neg s\rceil) \rightarrow \neg\lceil Says(M, \neg s) \Rightarrow \neg s\rceil$
$M_6 : Says(M, \neg s)$
$M_7 : M_4 \Rightarrow \neg s$
$W_0 : \rightarrow Says(WF, r)$
$W_1 : Says(WF, r) \Rightarrow r$
$JM : J_5, M_5 \rightarrow \neg r$

When applying the attack relation specified by Definition 15, the argumentation framework in Figure 2 can be built. In this argumentation framework, only one preferred extension exists: $\{J_0, J_1, J_2, M_0, M_1, M_2, W_0\}$*. So again, we have that the weather forecast is not justfied because unreliable John and unreliable Mary are having a disagreement about a cup of coffee.*

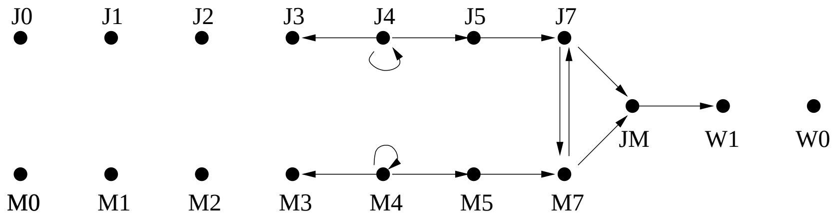

Figure 2: Preferred semantics does not always provide a solution

The idea of non-interference is that for two completely independent knowledge base $\mathcal{F}_1$ and $\mathcal{F}_2$, $\mathcal{F}_1$ does not influence the outcome with respect to the language of $\mathcal{F}_2$ and vice versa.

Postulate 4. *We say that a logical formalism* $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ *satisfies* non-interference *iff for every* $\mathcal{F}_1, \mathcal{F}_2 \subseteq \mathcal{F}ormulas$ *such that* $\mathcal{F}_1$ *and* $\mathcal{F}_2$ *are syntactically disjunct it holds that* $Cn(\mathcal{F}_1)_{|atoms(\mathcal{F}_1)} = Cn(\mathcal{F}_1 \cup \mathcal{F}_2)_{|atoms(\mathcal{F}_1)}$ *and* $Cn(\mathcal{F}_2)_{|atoms(\mathcal{F}_2)} = Cn(\mathcal{F}_1 \cup \mathcal{F}_2)_{|atoms(\mathcal{F}_2)}$*.*

For complete semantics of argumentation system, non-interference means that arguments do not affect the fact that whether a syntactically disjoint argument can be justified or not.

Definition 21. *A formalism for argument-based inference satisfies non-interference iff for any syntactically disjoint inference bases* $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ *and* $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ *it holds that*

1. $Cn(\mathcal{B})_{|\mathtt{Atoms}(\mathcal{B}_1)} = Cn(\mathcal{B}_1)$*, and*
2. $Cn(\mathcal{B})_{|\mathtt{Atoms}(\mathcal{B}_2)} = Cn(\mathcal{B}_2)$

where $\mathcal{B} = (\mathcal{P}_1 \cup \mathcal{P}_2, \mathcal{D}_1 \cup \mathcal{D}_2)$*.*

Some formulas can make the knowledge base that obtained by merging it with other set of formulas still yields the same outcome even the other set of formulas is completely independent. We call this phenomenon contamination.

Postulate 5. *Let* $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ *be a logical formalism. A set* $\mathcal{F}_1 \subseteq \mathcal{F}ormulas$*, with* $atoms(\mathcal{F}_1) \subsetneq \mathcal{A}toms$*, is called* contaminating *iff for every* $\mathcal{F}_2 \subseteq \mathcal{F}ormulas$ *such that* $\mathcal{F}_1$ *and* $\mathcal{F}_2$ *are syntactically disjunct it holds that* $Cn(\mathcal{F}_1) = Cn(\mathcal{F}_1 \cup \mathcal{F}_2)$*.*

Definition 22. *Let* $\mathcal{B} = (\mathcal{P}, \mathcal{D})$ *and* $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ *be two inference bases such that* $\mathrm{Atoms}(\mathcal{B}_1) \subsetneq \mathrm{Atoms}(\mathcal{P})$*. Then* $\mathcal{B}_1$ *is called* contaminating *iff for any inference base* $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ *that is syntactically disjoint with* $\mathcal{B}_1$ *and* $\mathrm{Atoms}(\mathcal{B}_2) \subseteq \mathrm{Atoms}(\mathcal{P})$*. It holds that* $Cn(\mathcal{B}_1) = Cn(\mathcal{B}_3)$ *where* $\mathcal{B}_3 = (\mathcal{P}_1 \cup \mathcal{P}_2, \mathcal{D}_1 \cup \mathcal{D}_2)$*.*

Crash resistance is defined based on the concept of contamination.

Postulate 6. *We say that a logical formalism satisfies* crash resistance *iff there does not exist a set of formulas $\mathcal{F}$ that is contaminating.*

A system that satisfies crash resistance can not be affected by totally unrelated factors.

Two sets of formulas that contain the same set of atoms could entail different results. For example, in classical logic, $Cn(\{a \wedge b\}) \neq Cn(\{a \vee b\})$.

Postulate 7. *We say that a logical formalism* $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ *is* non-trivial *iff for each $\mathcal{A} \subseteq \mathcal{A}toms$ such that $\mathcal{A} \neq \emptyset$ there exists $\mathcal{F}_1, \mathcal{F}_2 \subseteq \mathcal{F}ormulas$ such that $atoms(\mathcal{F}_1) = atoms(\mathcal{F}_2) = \mathcal{A}$ and $\mathcal{F}_2$ and $Cn(\mathcal{F}_1)_{|\mathcal{A}} \neq Cn(\mathcal{F}_2)_{|\mathcal{A}}$.*

Definition 23. *A formalism for argument-based inference satisfies* non-trivial *iff for each nonempty set $\mathcal{A}$ of atoms there exist inference bases $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ and $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ such that* $\text{Atoms}(\mathcal{B}_1) = \text{Atoms}(\mathcal{B}_2) = \mathcal{A}$ *it holds that $Cn(\mathcal{B}_1)_{|\mathcal{A}} \neq Cn(\mathcal{B}_2)_{|\mathcal{A}}$.*

For any non-trivial formalism, non-interference implies crash resistance.

Theorem 1. *Each non-trivial logical formalism $(\mathcal{A}toms, \mathcal{F}ormulas, Cn)$ that satisfies non-interference also satisfies crash resistance.*

Proof. Please refer to [2]. □

3 Solution

In this section we provide a solution to the problem of contamination of ASPIC system. In Figure 2, the argument JM contaminates the whole argumentation framework. It makes argument W_1 being affected by completely irrelevant information which is the reason of the contamination. We avoid this by destroying the inconsistent arguments. We build an argumentation framework AF from a defeasible theory. Then the argumentation framework AF' obtained by deleting all inconsistent arguments from AF is the result framework we want.

An argument is inconsistent if there are conclusions of its sub-arguments that are not consistent.

Definition 24 (consistent argument)**.** *Let $(\mathcal{P}, \mathcal{D})$ be an inference base and (Ar, def) the argumentation framework associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$. We say that an argument $A \in Ar$ is* consistent *iff* $\{\texttt{Conc}(A') \mid A' \in \texttt{SubArgs}(A)\}$ *is (propositionally) consistent. Otherwise, the argument is* inconsistent*.*

Definition 25. *Let (Ar, def) be an argumentation framework built from a defeasible theory $\mathcal{T} = (\mathcal{S}, \mathcal{D})$ and $\mathcal{S}$ be consistent. Let $A \in Ar$ be an inconsistent argument and let $A_1, \ldots, A_n \in \texttt{Sub}(A)$. We say $\{A_1, \ldots, A_n\}$ is an* inconsistent base *of A if $\{\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n)\}$ is a minimal inconsistent set of conclusions of A.*

The approach that is proposed in the current paper is to delete any inconsistent arguments from the argumentation framework, before applying the abstract argumentation semantics.

Definition 26. *Let $(\mathcal{P}, \mathcal{D})$ be an inference base and (Ar, def) be the associated argumentation framework associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$. We define Ar_c as $\{A \mid A \in Ar$ and A is consistent $\}$, and $def_c = def \cap (Ar_c \times Ar_c)$. We refer to (Ar_c, def_c) as the inconsistency cleaned argumentation framework associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$.*

Although it is possible to construct inconsistent arguments from a defeasible theory, they will never be in any complete extensions.

Lemma 1. *Let $(\mathcal{P}, \mathcal{D})$ be an inference base. Let $AF = (Ar, def)$ be an argumentation framework built from $\mathcal{T} = (\mathcal{S}(\mathcal{P}), \mathcal{D})$. Let $A \in Ar$ and A be an inconsistent argument. For all E such that E is a complete extension of AF, $A \notin E$.*

Proof. Assume that there exists a complete extension E of AF such that $A \in E$ and A is an inconsistent argument. Let $\{A'_1, \ldots, A'_m\}$ be an inconsistent base of A. There exists an argument B such that TopRule(B) is a defeasible rule and $B \in \text{Sub}(A'_1) \cup \ldots \cup \text{Sub}(A'_m)$ because $\mathcal{S}$ is consistent. $\{A'_1, \ldots, A'_m, B\} \subseteq E$ because $A'_1, \ldots, A'_m, B \in \texttt{Sub}(A)$ and from the fact that E is a complete extension implies that it is closed under subarguments. So E defends $A'_1, \ldots, A'_m$ and B . Let Conc$(B) = c$. Then we can construct an argument $A' = A'_1, \ldots, A'_m \to -c$. For all $1 \leq i \leq m$, $A'_i \in E$ Then E defends A'. Therefore $A' \in E$. Then A' defeats B. So E is not conflict-free. Contradiction. □

After we delete the inconsistent arguments from the argumentation framework, closure is still satisfied.

Theorem 2. *Let $(\mathcal{P}, \mathcal{D})$ be an inference base. Every complete extension of the inconsistency cleaned argumentation framework (Ar_c, def_c) associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$ is closed.*

Proof. Let E be a complete extension. Suppose that $\{\texttt{Conc}(A) \mid A \in E\} \neq Cl_{\mathcal{S}}(\{\texttt{Conc}(A) \mid A \in E\})$. This means that there exist arguments $A_1, \ldots, A_n \in E$ with $\texttt{Conc}(A_1) = \phi_1, \ldots, \texttt{Conc}(A_n) = \phi_n$ and $\exists \phi_1, \ldots, \phi_n \to \psi \in \mathcal{S}(\mathcal{P})$, but $A = A_1, \ldots, A_n \to \psi \notin E$. Three possible cases exist:

Case 1: $E \cup \{A\}$ is not conflict-free. Then either $\exists B \in E$ such that B defeats A, or $\exists B \in E$ such that A defeats B.

Suppose that $\exists B \in E$ such that B defeats A on a subargument A'. Thus, $A' \in \texttt{Sub}(A)$. However, $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$. According to the definition of restricted rebutting and that of undercut, the top rule of A' is defeasible. Thus, $A' \in \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n)$. Then $A' \in \texttt{Sub}(A_1)$, or ..., or $A' \in \texttt{Sub}(A_n)$. Then $A' \in E$. Thus E is not conflict-free. Contradiction.

Suppose that $\exists B \in E$ such that A defeats B. As E is an admissible set, it must defend itself against A. This can only be the case if E contains some argument C such that C defeats A_1 or ... or A_n. But then E would not be conflict-free. Contradiction.

Case 2: E does not defend A. This means that $\exists B \in Ar$ such that B defeats A and $\neg \exists C \in E$ such that C defeats B. Since B defeats A, it must hold that B rebuts or undercut A on a subargument A' whose top rule is defeasible. Thus B rebuts or undercut A'. Since $A' \in \texttt{Sub}(A)$ and $A' \neq A$, it holds that $A' \in \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n)$. Then $A' \in E$. Consequently, A' is defended by E against B. Contradiction.

Case 3: $E \cup \{A\}$ is a complete extension and A is an inconsistent argument. So A is deleted from the argumentation framework. From Lemma 1, it is not possible. $\square$

We now show that if we get rid of the inconsistent arguments then direct consistency and indirect consistency are satisfied under each of Dung's standard semantics.

Theorem 3. *Let $(\mathcal{P}, \mathcal{D})$ be an inference base. Every complete extension of the inconsistency cleaned argumentation framework (Ar_c, def_c) associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$ satisfies direct consistency.*

Proof. Suppose some complete extension Ar of (Ar_c, def_c) does not satisfy direct consistency. Then, by definition, Ar contains an argument (say A) with conclusion ψ and an argument (say B) with conclusion $\neg\psi$. As $\mathcal{P}$ is assumed to be consistent, it must hold that A or B contains at least one defeasible rule (say d). Now, consider the strict rule $\psi, \neg\psi \rightarrow -\texttt{Conc}(d)$, which is in $S(\mathcal{P})$. As the conclusions of Ar satisfy closure (Theorem 2), $-\texttt{Conc}(d)$ must be a conclusion of Ar. But that would mean that there is some argument (say C) in Ar with conclusion $-\texttt{Conc}(d)$. But then Ar would not be conflict-free. Contradiction. $\square$

Theorem 4. *Let $(\mathcal{P}, \mathcal{D})$ be an inference base. Every complete extension of the inconsistency cleaned argumentation framework (Ar_c, def_c) associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$ satisfies indirect consistency*

Proof. It follows from Proposition 2 and Theorem 3. $\square$

Lemma 2. *Let At_1 and At_2 be sets of atoms such that $At_1 \cap At_1 = \emptyset$. Let Ψ_1 and Ψ_2 be sets of formulas with $\texttt{Atoms}(\Psi_1) \subseteq At_1$ and $\texttt{Atoms}(\Psi_2) \subseteq At_2$. And let ϕ be a formula with $\texttt{Atoms}(\phi) \subseteq At_1$ and $\Psi_1, \Psi_2 \vdash \phi$. Then $\Psi_1 \vdash \phi$ or Ψ_2 is inconsistent.*

We will show that each consistent argument A that contains atoms from both of the syntactically disjoint defeasible theories can be mapped into an argument A' that contains atoms of only one defeasible theory. These two arguments defeat the same arguments because their conclusions are same. If an argument defeats A' then it also defeats A. If a set of arguments defends A then it defends A'.

Definition 27. *Let $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ and $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ be two syntactically disjoint inference bases. Let (Ar_1, def_1) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_1 = (\mathcal{S}(\mathcal{P}_1), \mathcal{D}_1)$ and (Ar_2, def_2) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_2 = (\mathcal{S}(\mathcal{P}_2), \mathcal{D}_2)$. Let (Ar, def) be the inconsistency cleaned argumentation framework associated with $\mathcal{T} = (\mathcal{S}(\mathcal{P}_1) \cup \mathcal{S}(\mathcal{P}_2), \mathcal{D}_1 \cup \mathcal{D}_2)$. Let $A \in Ar$ and $A = A_1, \ldots, A_n, B_1, \ldots, B_m \rightsquigarrow c$ $(n, m \geq 0$ and $\rightsquigarrow \in \{\rightarrow, \Rightarrow\})$. Let $\texttt{Atoms}(\texttt{Conc}(A)) \subseteq \texttt{Atoms}(\mathcal{B}_1)$, $\texttt{Atoms}(\texttt{Conc}(A_1)) \cup \ldots \cup \texttt{Atoms}(\texttt{Conc}(A_n)) \subseteq \texttt{Atoms}(\mathcal{B}_1)$ and $\texttt{Atoms}(\texttt{Conc}(B_1)) \cup \ldots \cup \texttt{Atoms}(\texttt{Conc}(B_m)) \subseteq \texttt{Atoms}(\mathcal{B}_2)$. Then let $\mathcal{H} : Ar \mapsto Ar_1$ be a function such that:*

$$\mathcal{H}(A) = \begin{cases} A_1, \ldots, A_n \rightsquigarrow c & \text{if } \texttt{depth}(A) \leq 2 \\ \mathcal{H}(A_1), \ldots, \mathcal{H}(A_n) \rightsquigarrow c & \text{if } \texttt{depth}(A) > 2. \end{cases}$$

Lemma 3. *Let $\mathcal{H}(A) = A'$ and $A' \in Ar_1$. Then $A^{'-} \subseteq A^-$ and $A^+ = A^{'+}$.*

Proof. Let $B \in A^{'-}$. Then B defeats A' on a defeasible rule r. There are two cases:

1. Let $\texttt{Conc}(r) = c$. Then $\texttt{Conc}(B) = \neg c$. A also contains r. Therefore $B \mathit{def} A$. So $B \in A^-$.

2. Let $\texttt{Conc}(r) = d$ where d is a defeasible rule. Then $\texttt{Conc}(B) = \neg\lceil d\rceil$. A also contains r. Therefore $B \mathit{def} A$. So $B \in A^-$.

$A^+ = A^{'+}$ because $\texttt{Conc}(A) = \texttt{Conc}(A')$.

□

Lemma 4. *Let $\mathcal{H}(A) = A'$ and $\mathcal{A}rgs$ be a set of arguments. If $A \in F(\mathcal{A}rgs)$ then $A' \in F(\mathcal{A}rgs)$.*

Proof. Suppose that $A \in F(\mathcal{A}rgs)$. For each B that defeats A', $B \mathit{def} A$ according to Lemma 3. Then each B that defeats A' is defeated by arguments in $\mathcal{A}rgs$ because $A \in F(\mathcal{A}rgs)$. □

In the rest of the paper we use F represent the function in Definition 2 under AF, F_1 under AF_1 and F_2 under AF_2.

The following Lemma shows that if E_1 and E_2 are complete extensions of two argumentation frameworks AF_1 and AF_2 that were built from two syntactically disjoint defeasible theories, then the arguments in AF_1 (AF_2) that are defended by the union of the two complete extensions under the argumentation framwork built from the union of the two defeasible theories is exactly E_1 (E_2).

Lemma 5. *Let $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ and $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ be two syntactically disjoint inference bases. Let (Ar_1, def_1) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_1 = (\mathcal{S}(\mathcal{P}_1), \mathcal{D}_1)$ and (Ar_2, def_2) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_2 = (\mathcal{S}(\mathcal{P}_2), \mathcal{D}_2)$. Let (Ar, def) be the inconsistency cleaned argumentation framework associated with $\mathcal{T} = (\mathcal{S}(\mathcal{P}_1) \cup \mathcal{S}(\mathcal{P}_2), \mathcal{D}_1 \cup \mathcal{D}_2)$. Let E_1 and E_2 be complete extensions of AF_1 and AF_2 respectively. Then*

1. $Ar_1 \cap F(E_1 \cup E_2) = E_1$ *and*

2. $Ar_2 \cap F(E_1 \cup E_2) = E_2$.

Proof. We now prove the first property (the proof of the second property is similar).

"$\subseteq$": Let $A \in Ar_1$ and $A \in F(E_1 \cup E_2)$. Assume that $A \notin E_1$. Then $A \notin F_1(E_1)$ since $E_1 = F_1(E_1)$. So $\exists B \in Ar_1 . B \mathit{def} A$ such that $\neg\exists C \in E_1 . C \mathit{def} B$. From the fact that $A \in F(E_1 \cup E_2)$ it follows that for all $B' \in Ar$ such that $B' \mathit{def} A$ there exists $C' \in E_1 \cup E_2$ such that $C' \mathit{def} B'$. Then there exists an argument $C' \in E_1 \cup E_2$ such that $C' \mathit{def} B$. Then $C' \in E_2$ because $\neg\exists C \in E_1 . C \mathit{def} B$. From the fact that $B \in Ar_1$ and $\texttt{Atoms}(Ar_1) \cap \texttt{Atoms}(Ar_2) = \emptyset$ it follows that for all C' such that $C' \mathit{def} B$, $C' \notin Ar_2$ because inconsistent arguments have been deleted. So $C' \notin E_2$. Contradiction.

"$\supseteq$": Let $A \in E_1$. Assume that $A \notin F(E_1 \cup E_2)$. Then there exists an argument $B \in Ar$ such that $B \mathit{def} A$ and there exists no argument $C \in E_1 \cup E_2$

such that $C\,def\,B$. Then $\texttt{Atoms}(\texttt{Conc}(B)) \subseteq \texttt{Atoms}(Ar_1)$. Let $B' = \mathcal{H}(B)$. Then $B'\,def\,A$ from Lemma 3 and $B' \in Ar_1$. Then there exists an argument $C' \in E_1$ such that $C'\,def\,B' \in \mathcal{T}_1$ because E_1 is a complete extension of AF_1 and $A \in E_1$. It follows that $C'\,def\,B \in \mathcal{T}$. Contradiction. □

In the inconsistency cleaned version of the ASPIC system, complete semantics satisfies non-interference.

Lemma 6. *Let $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ be two sets of arguments and $\mathcal{A}rgs_1 = \mathcal{A}rgs_2$. Then* $\text{Concs}(Args_1) = \text{Concs}(\mathcal{A}rgs_2)$.

Proof. "$\subseteq$": Let $c \in \text{Concs}(\mathcal{A}rgs_1)$. Then $\exists A \in \mathcal{A}rgs_1$ such that $\text{Conc}(A) = c$. So $A \in \mathcal{A}rgs_2$ because $\mathcal{A}rgs_1 = \mathcal{A}rgs_2$. Then $c \in \text{Concs}(\mathcal{A}rgs_2)$.

"$\supseteq$": Let $c \in \text{Concs}(\mathcal{A}rgs_2)$. Then $\exists A \in \mathcal{A}rgs_2$ such that $\text{Conc}(A) = c$. So $A \in \mathcal{A}rgs_1$ because $\mathcal{A}rgs_1 = \mathcal{A}rgs_2$. Then $c \in \text{Concs}(\mathcal{A}rgs_1)$.

□

Lemma 7. *Let $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ and $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ be two syntactically disjoint inference bases. Let (Ar_1, def_1) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_1 = (\mathcal{S}(\mathcal{P}_1), \mathcal{D}_1)$ and (Ar_2, def_2) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_2 = (\mathcal{S}(\mathcal{P}_2), \mathcal{D}_2)$. Let $AF = (Ar, def)$ be the inconsistency cleaned argumentation framework associated with $\mathcal{T} = (\mathcal{S}(\mathcal{P}_1) \cup \mathcal{S}(\mathcal{P}_2), \mathcal{D}_1 \cup \mathcal{D}_2)$. Let E be a complete extension of AF. Then* $\text{Concs}(E \cap Ar_1) = \text{Concs}(E)_{|\text{Atoms}(Ar_1)}$.

Proof. "$\subseteq$": Let $c \in \text{Concs}(E \cap Ar_1)$. Then $\exists A \in E \cap Ar_1$ such that $\text{Conc}(A) = c$. So $A \in E$ and $A \in Ar_1$. Then $c \in \text{Concs}(E)$ and $c \in \text{Concs}(Ar_1)$. So $\text{Atoms}(c) \subseteq \text{Atoms}(Ar_1)$. Therefore $c \in \text{Concs}(E)_{|\text{Atoms}(Ar_1)}$.

"$\supseteq$": Let $c \in \text{Concs}(E)_{|\text{Atoms}(Ar_1)}$. Then $\exists A \in E$ such that $\text{Conc}(A) = c$ and $\text{Atoms}(c) \subseteq \text{Atoms}(Ar_1)$. Then there exists an argument $A' = \mathcal{H}(A)$ and $A' \in Ar_1$ because $\text{Atoms}(c) \in \text{Atoms}(Ar_1)$. $A' \in E$ because E defends A' according to Lemma 4. Therefore $A' \in E \cap Ar_1$. Then $\text{Conc}(A') \in \text{Concs}(E \cap Ar_1)$. $\text{Conc}(A') = \text{Conc}(A) = c$ because $A' = \mathcal{H}(A)$. Hence, $c \in \text{Concs}(E \cap Ar_1)$.

□

Theorem 5. *Let Cn be a function that takes an inference base as its input and produces a set $\{\texttt{Concs}(\mathcal{A}rgs_1), \ldots, \texttt{Concs}(\mathcal{A}rgs_n)\}$, where $\mathcal{A}rgs_1, \ldots, \mathcal{A}rgs_n$ $(n \geq 1)$ are the complete extensions of the inconsistency cleaned argumentation framework associated with $(\mathcal{S}(\mathcal{P}), \mathcal{D})$. It holds that Cn satisfies non-interference.*

Proof. Let $\mathcal{B}_1 = (\mathcal{P}_1, \mathcal{D}_1)$ and $\mathcal{B}_2 = (\mathcal{P}_2, \mathcal{D}_2)$ be two syntactically disjoint inference bases. Let (Ar_1, def_1) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_1 = (\mathcal{S}(\mathcal{P}_1), \mathcal{D}_1)$ and (Ar_2, def_2) be the inconsistency cleaned argumentation framework associated with $\mathcal{T}_2 = (\mathcal{S}(\mathcal{P}_2), \mathcal{D}_2)$. Let (Ar, def) be the inconsistency cleaned argumentation framework associated with $\mathcal{T} = (\mathcal{S}(\mathcal{P}_1) \cup \mathcal{S}(\mathcal{P}_2), \mathcal{D}_1 \cup \mathcal{D}_2)$.

Let $Ar_{comb} \subseteq Ar$ such that $Ar_{comb} = \{A \in Ar \mid \texttt{Atoms}(A) \cap \texttt{Atoms}(AF_1) \neq \emptyset$ and $\texttt{Atoms}(A) \cap \texttt{Atoms}(AF_2) \neq \emptyset\}$. Then $Ar_1 \cap Ar_2 = \emptyset$, $Ar_1 \cap Ar_{comb} = \emptyset$, $\mathcal{A}_2 \cap Ar_{comb} = \emptyset$ and $Ar = Ar_1 \cup \mathcal{A}_2 \cup Ar_{comb}$.

In order to show non-interference, we have to show that:

1. $Cn_{complete}(\mathcal{B}_1)_{|atoms(\mathcal{B}_1)} = Cn_{complete}(\mathcal{B})_{|atoms(\mathcal{B}_1)}$, and
2. $Cn_{complete}(\mathcal{B}_2)_{|atoms(\mathcal{B}_2)} = Cn_{complete}(\mathcal{B})_{|atoms(\mathcal{B}_2)}$.

According to Lemma 6 and Lemma 7, it is sufficient to prove that
1. $\text{CompExt}(\mathcal{B}_1)_{|atoms(\mathcal{B}_1)} = \text{CompExt}(\mathcal{B})_{|atoms(\mathcal{B}_1)}$, and
2. $\text{CompExt}(\mathcal{B}_2)_{|atoms(\mathcal{B}_2)} = \text{CompExt}(\mathcal{B})_{|atoms(\mathcal{B}_2)}$.
Because $\text{Concs}(\text{CompExt}(\mathcal{B}_1)_{|atoms(Ar_1)}) = Cn_{complete}(\mathcal{B}_1)_{|atoms(Ar_1)}$ and $\text{Concs}(\text{CompExt}(\mathcal{B})_{|atoms(Ar_1)}) = Cn_{complete}(\mathcal{B})_{|atoms(Ar_1)}$ according to Lemma 7, if we can prove that $\text{CompExt}(\mathcal{B}_1)_{|atoms(Ar_1)} = \text{CompExt}(\mathcal{B})_{|atoms(Ar_1)}$ then $Cn_{complete}(\mathcal{B}_1)_{|atoms(Ar_1)} = Cn_{complete}(\mathcal{B})_{|atoms(Ar_1)}$ holds according to Lemma 6.

We now prove the first property (the proof of the second property is similar).

"$\subseteq$": Let E_1 be a complete extension of AF_1. We now have to prove that there exists a complete extension E of AF such that $E \cap Ar_1 = E_1$.

Let E_2 be a complete extension of AF_2. Let $E = F(E_1 \cup E_2)$ and let $E_c = E \cap Ar_{comb}$. From Lemma 5 we know that $E_1 = E \cap Ar_1$ and $E_2 = E \cap Ar_2$. Then $E = E_1 \cup E_2 \cup E_c$. We now prove that E is a complete extension of AF.

First we prove that E is conflict-free. $E_1 \cup E_2$ is conflict-free since no argument in E_1 attacks any argument in E_2 and vice versa (this is because AF_1 and AF_2 are syntactically disjoint). We need to prove that

1. There is no argument in E_1 that defeats any arguments in E_c and vice versa.
2. There is no argument in E_2 that defeats any arguments in E_c and vice versa.
3. E_c is conflict-free.

We now prove the first property (the proof of the second property is similar).

(1) Assume that $\exists A \in E_1$ and $\exists B \in E_c$ such that $A \mathit{def} B$. Then $B \in F(E_1 \cup E_2)$. So $\exists C \in E_1 \cup E_2$ such that $C \mathit{def} A$. Then $C \notin Ar_2$ otherwise C can not defeat A because A_1 and A_2 are syntactically disjoint and inconsistent arguments have been deleted. So $C \in E_1$. Then E_1 is not conflict-free. Contradiction.

(2) Assume that $\exists A \in E_c$ and $\exists B \in E_1$ such that $A \mathit{def} B$. Let $A' = \mathcal{H}(A)$. Then $A' \in Ar_1$ and $A' \mathit{def} B$ according to Lemma 3. Then from the fact that E_1 is admissible, it follows that there exists an argument $C \in E_1$ such that $C \mathit{def} A'$. $C \mathit{def} A$ from Lemma 3. From the fact that $A \in F(E_1 \cup E_2)$ it follows that there exists an argument $D \in E_1 \cup E_2$ such that $D \mathit{def} C$. So $D \in Ar_1 \cup Ar_2$. Then $D \in Ar_1$ because $C \in Ar_1$. So $D \in E_1$. Then E_1 is not conflict-free. Contradiction.

Now we prove the third property. Assume that E_c is not conflict-free. Then $\exists A, B \in E_c$ such that $A \mathit{def} B$. From the fact that $B \in F(E_1 \cup E_2)$ it follows that $\exists C \in E_1 \cup E_2$ such that $C \mathit{def} A$. Then there exists an argument in $E_1 \cup E_2$ defeats an argument in E_c. It contradicts with (1) and (2).

The next thing to prove is that E is a fixpoint of F under AF.

$E \subseteq F(E)$:: Let $A \in E$. Then $A \in F(E_1 \cup E_2)$. So $\forall B \in Ar$ such that $B \mathit{def} A$ there exists an argument $C \in E_1 \cup E_2$ such that $C \mathit{def} B$. Then $C \in E$ because $E_1 \cup E_2 \subseteq E$. Therefore $\forall B \in Ar$ such that $B \mathit{def} A$ there exists an argument $C \in E$ such that $C \mathit{def} B$. So $A \in F(E)$.

$F(E) \subseteq E$:: Let $A \in F(E)$. Then $\forall B \in Ar$ such that $B\,def\,A$ there exists an argument $C \in E_1 \cup E_2 \cup E_c$ such that $C\,def\,B$. Assume that $A \notin E$. Then $A \notin F(E_1 \cup E_2)$. So there exists an argument $B' \in Ar$ that $B'\,def\,A$ and there exists no argument $C \in E_1 \cup E_2$ such that $C\,def\,B'$. Then it follows that there exists an argument $C' \in E_c$ such that $C'\,def\,B'$. Let $C'' = \mathcal{H}(C')$. Then $C'' \in Ar_1 \cup Ar_2$ and $C''\,def\,B'$. $C' \in F(E_1 \cup E_2)$ because $C' \in E_c$ and $E_c \subseteq F(E_1 \cup E_2)$. Then $C'' \in F(E_1 \cup E_2)$ from Lemma 4. It follows that $C'' \in F(E_1 \cup E_2) \cap (Ar_1 \cup Ar_2)$. Therefore $C'' \in E_1 \cup E_2$ according to Lemma 5. Contradiction.

From the fact that E is a conflict-free set with $E \subseteq F(E)$ and $F(E) \subseteq E$ it then follows that E is a complete extension of AF.

"$\supseteq$" Let E be a complete extension of AF and let $E_1 = E \cap Ar_1$ and $E_c = E \cap Ar_{comb}$. We now have to prove that E_1 is a complete extension AF_1. The fact that E_1 is conflict-free follows from the fact that E is conflict-free.We now prove that E_1 is a fixpoint of F_1 under AF_1.

$E_1 \subseteq F_1(E_1)$:: Let $A \in E_1$. Then from the fact that $E_1 \subseteq E$ it follows that $A \in E$. From the fact that E is a complete extension it follows that $A \in F(E)$. That is, for each B that attacks A, there exists a $C \in E$ that attacks B. Assume that $A \notin F_1(E_1)$. Then there exists a $B' \in Ar_1$ that $B'\,def\,A$ and there exists no $C \in E_1$ that $C\,def\,B'$. Then there exists a $C' \in E_c$ that $C'\,def\,B'$. Let $C'' = \mathcal{H}(C')$. Then $C'' \in Ar_1$ and $C''\,def\,B'$ (Lemma 3). $C' \in E$ because $C' \in E_c$ and $E_c \subseteq S$. From the fact that $E = F(E)$ it follows that $C' \in F(E)$. From Lemma 4, $C'' \in F(E)$. Then $C'' \in E$. $C'' \in E \cap Ar_1$ because $C'' \in E$ and $C'' \in Ar_1$. Therefore $C'' \in E_1$. Contradiction.

$F_1(E_1) \subseteq E_1$:: Let $A \in Ar_1$ and $A \in F_1(E_1)$. Then for each $B \in Ar_1$ that attacks A there exists a $C \in E_1$ that attacks B. Assume that $A \notin E_1$. Then $A \notin E$. So $A \notin F(E)$. Then there exists a $B' \in Ar$ that $B'\,def\,A$ and there exists no $C \in E$ that $C\,def\,B'$. $B' \notin Ar_2$ because $A \in Ar_1$ and A_1 and A_2 are syntactically disjoint. So there are two cases:

(1) $B' \in Ar_1$. From the fact that $A \in F_1(E_1)$ it follows that there exists a $C \in E_1$ that $C\,def\,B'$. So there exists a $C \in E$ that $C\,def\,B'$ because $E_1 \subseteq E$. Contradiction.

(2) $B' \in Ar_{comb}$. Let $B'' = \mathcal{H}(B')$. Then $B'' \in Ar_1$ and $B''\,def\,A$ according to Lemma 3. So there exists a $C \in E_1$ that $C\,def\,B''$ because $A \in F(E_1)$. Therefore $C\,def\,B'$ (Lemma 3). Contradiction.

From the fact that E_1 is a conflict-free set with $E_1 \subseteq F_1(E_1)$ and $F_1(E_1) \subseteq E_1$ it then follows that E_1 is a complete extension of AF_1. □

The complete semantics satisfies non-trivial.

Theorem 6. *In ASPIC system, complete semantics is non-trivial.*

Proof. Let $\mathcal{A}t$ be a non-empty set of atoms. We have to prove that there exist two argumentation frameworks $AF_1 = (\mathcal{A}_1, def_1)$ and $AF_2 = (\mathcal{A}_2, def_2)$ built from $\mathcal{T}_1$ and $\mathcal{T}_2$ respectively such that $\text{Atoms}(\mathcal{T}_1) = \text{Atoms}(\mathcal{T}_2) = \mathcal{A}t$ and $\mathcal{C}n_{complete}(AF_1) \neq \mathcal{C}n_{complete}(AF_2)$. This is obtained with $\mathcal{T}_1 = (\mathcal{P}_1, \mathcal{D}_1) = (\{\rightarrow a\}, \emptyset)$ and $\mathcal{T}_2 = (\mathcal{P}_2, \mathcal{D}_2) = (\{a \rightarrow a\}, \emptyset)$ where $a \in \mathcal{A}t$. In that case, $\mathcal{C}n_{complete}(AF_1) = \{a\}$ and $\mathcal{C}n_{complete}(AF_2) = \emptyset$. □

From the fact that any non-trivial formalism, non-interference implies crash resistance, it follows that cin ASPIC system, complete semantics satisfies crash resistance.

Theorem 7. *In ASPIC system, complete semantics satisfies crash resistency.*

Proof. It follows from Theorem 5 and Theorem 6. □

Now let's consider Example 1 using the new principle.

Example 2. *Consider the arguments in Example 1. The argumen JM is deleted from the argumentation framework because it is an inconsistent argument. Then we got the new argumentation framework in Figure 3. In this new argumentation framework, W_1 is in the grounded extension and the preferred extension. The forecast is no longer affected by a quarrel between other two persons on a cup of coffee.*

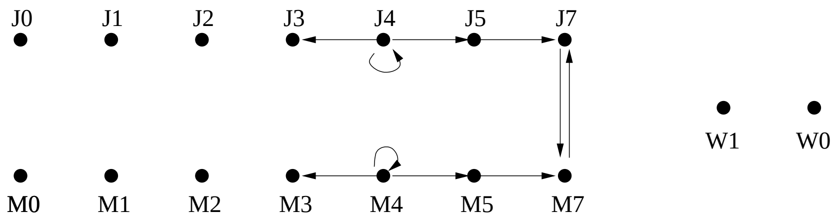

Figure 3: Repaired argumentation framework

4 Discussion

In this paper we provide a solution for ASPIC system to avoid being affected by contaminating information.

Argumentation frameworks are built from defeasible theories. Some defeasible theories can be divided into syntactically disjoint parts which produce completely unconnected graphs. The consequences of the whole argumentation framework should simply be the union of the consequence of each sub-framework because they are logically unrelated. The inconsistent arguments can connect the graphs together and change the consequence of the frameworks. It makes the unrelated arguments affect each other so that the consequence becomes unreasonable. In order to solve this problem we delete inconsistent arguments from the argumentation frameworks. Then the complete semantics in argumentation framework satisfies the five postulates, closure, direct consistency, indirect consistency, non-interference and crash resistance. Those postulates insure that the system will not crash and can produce logically reasonable results when contaminating information are being input. Since the five postulates are satisfied by complete semantics, then they are also satisfied by grounded, preferred, semi-stable and stable semantics. So the problems that are in [13] and [10] are solved.

We treat a relatively simple formalism given in [1] in this paper. For one further study, we can consider a more complex formalism. For instance, the formalism in [13]. Deleting inconsistent arguments might lose some information. So another possible further study could be another solution that can keep as much information as possible for the same problems.

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