An Enriched Argumentation Framework with Higher-Level Relations

Agree to Disagree 2020

Jérémie Dauphin & Marcos Cramer

University of Luxembourg

November 10, 2020

Outline

Outline

1. Introduction
2. Labeling EAFs
3. Extended Explanatory Argumentation Frameworks
   - The framework
   - The relations
   - Labeling the framework
4. Correspondence with flattening
5. Conclusion

Introduction

- Formal Argumentation: Study of reasoning via arguments.
- Abstract Argumentation: Focus on argument relations.
- Many enrichments for Abstract Argumentation Frameworks.
- How do we aggregate them while preserving sensible evaluation?

Explanatory Argumentation Frameworks[1]

Explanatory Argumentation Frameworks

An *explanatory argumentation framework* (EAF) is a tuple $\langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$, where:

- $\mathcal{A}$: arguments;
- $\mathcal{X}$: explananda;
- $\rightarrow \subseteq \mathcal{A} \times \mathcal{A}$: attacks;
- $\dashrightarrow \subseteq \mathcal{A} \times (\mathcal{A} \cup \mathcal{X})$: explanations;
- $\sim \subseteq \mathcal{A} \times \mathcal{A}$: incompatibility.

[1] introduced by Šešelja and Straßer in 2013

Admissibility in EAFs

Conflict-free

$S \subseteq \mathcal{A}$ is *conflict-free* iff there are no $a, b \in S$ s.t. $a \to b$ or $a \sim b$.

Defense

$S \subseteq \mathcal{A}$ *defends* $a \in \mathcal{A}$ iff for each $b \to a$, there is a $c \in S$ s.t. $c \to b$.

Admissible

$S \subseteq \mathcal{A}$ is admissible iff it is conflict-free and defends all its elements.

Semantics

Explanatory power

$S >_p S'$ iff S explains more explananda than S'.

Explanatory depth

$S >_d S'$ iff S has longer chains of explanations leading to an explananda than S'.

Semantics

Explanatory power

$S >_p S'$ iff S explains more explananda than S'.

Explanatory depth

$S >_d S'$ iff S has longer chains of explanations leading to an explananda than S'.

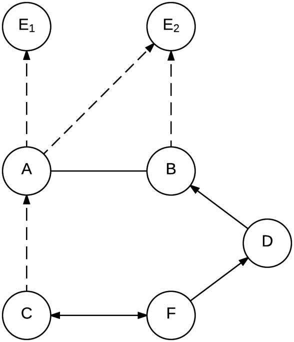
Argumentative core extensions

S is an AC-extension iff it is an admissible set which is maximal w.r.t $>_p$ and $\subseteq$.

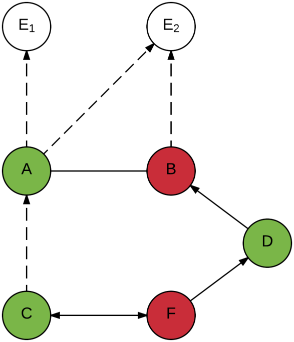
Explanatory core extensions

S is an EC-extension iff it is an admissible set which is maximal w.r.t $>_p$ and $>_d$ while being minimal w.r.t $\subseteq$.

Example

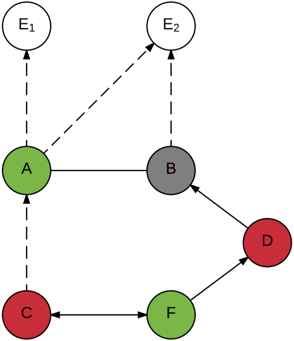


Example



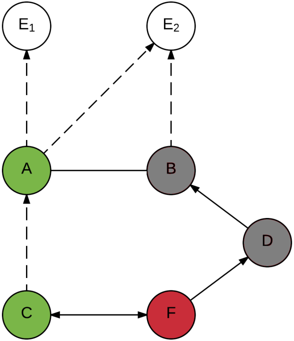
AC-extensions: $\{A, C, D\}, \{A, F\}$
EC-extensions: $\{A, C\}$

Example



AC-extensions: $\{A, C, D\}, \{A, F\}$
EC-extensions: $\{A, C\}$

Example



AC-extensions: $\{A, C, D\}, \{A, F\}$
EC-extensions: $\{A, C\}$

Outline

EAF labelings

Labeling function

Given an $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$.
Two part labeling function $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$:

- $\mathtt{Lab}_{\mathcal{A}}$: from $\mathcal{A}$ to $\{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$
- $\mathtt{Lab}_{\dashrightarrow}$: from $\dashrightarrow$ to $\{\mathtt{exp}, \mathtt{nexp}\}$

Legal labels

Legal labels

Given $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ an EAF, $b \in \mathcal{A}$ and $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$ a labeling of F. We say that b is (w.r.t. Lab):

- *legally* in iff $\forall c$ s.t. $c \rightarrow b$, $\mathtt{Lab}_{\mathcal{A}}(c) = \mathtt{out}$ and $\forall d$ s.t. $d \sim b$, $\mathtt{Lab}_{\mathcal{A}}(d) \neq \mathtt{in}$;
- *legally* out iff $\exists c$ s.t. $c \rightarrow b$ and $\mathtt{Lab}_{\mathcal{A}}(c) = \mathtt{in}$;
- *legally* undec otherwise;

For $(b, x) \in \dashrightarrow$, we say that (b, x) is (w.r.t Lab):

- *legally* exp iff $\mathtt{Lab}_{\mathcal{A}}(b) = \mathtt{in}$;
- *legally* nexp otherwise.

Admissible EAF labeling

Admissible EAF labeling

Given an EAF F and labeling `Lab`, we say that `Lab` is an *admissible labeling of F* iff:

- every `in` and `out` labelled argument is legally so;
- every `exp` and `nexp` labelled explanation arrow is legally so.

Admissible EAF labeling

Admissible EAF labeling

Given an EAF F and labeling `Lab`, we say that `Lab` is an *admissible labeling of* F iff:

- every `in` and `out` labelled argument is legally so;
- every `exp` and `nexp` labelled explanation arrow is legally so.

Complete EAF labeling

We say that `Lab` is a *complete labeling of* F iff `Lab` is an admissible labeling of F and additionally each `undec` labelled argument is legally so.

AC-labeling

AC-labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF and $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ a labeling of F. We say that $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ is an *argumentative core labeling (AC-labeling)* of F iff:

- $\texttt{Lab}$ is a $\subseteq$-maximal complete labeling of F;
- the set of explananda target of $\texttt{exp}$ labelled explanations is maximal.

Relevance and satisfactory labeling

Explanatory relevance

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF, let $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ a labeling of F, and let $(x, y) \in \dashrightarrow$. We say that (x, y) *has explanatory relevance w.r.t. $\mathtt{Lab}$* iff $\texttt{Lab}_{\dashrightarrow}((x, y)) = \texttt{exp}$ and there is a path of $\texttt{exp}$ labelled explanations which leads to an explananda.

Relevance and satisfactory labeling

Explanatory relevance

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF, let $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ a labeling of F, and let $(x, y) \in \dashrightarrow$. We say that (x, y) *has explanatory relevance w.r.t.* Lab iff $\texttt{Lab}_{\dashrightarrow}((x, y)) = \texttt{exp}$ and there is a path of $\texttt{exp}$ labelled explanations which leads to an explananda.

Satisfactory labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF and $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ a labeling of F. We say that $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ is a *satisfactory labeling* of F iff $\texttt{Lab}$ is an admissible labeling of F and there is no admissible labeling $\texttt{Lab}' = (\texttt{Lab}'_{\mathcal{A}}, \texttt{Lab}'_{\dashrightarrow})$ of F such that $\{e \in \mathcal{X} \mid \text{for some } b \in \mathcal{A}, \texttt{Lab}'_{\dashrightarrow}((b, e)) = \texttt{exp}\} \supsetneq \{e \in \mathcal{X} \mid \text{for some } b \in \mathcal{A}, \texttt{Lab}_{\dashrightarrow}((b, e)) = \texttt{exp}\}$.

EC-labeling

Insightful labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF and $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ a labeling of F. We say that $\texttt{Lab} = (\texttt{Lab}_{\mathcal{A}}, \texttt{Lab}_{\dashrightarrow})$ is an *insightful labeling* of F iff $\texttt{Lab}$ is an satisfactory labeling of F and there is no satisfactory labeling $\texttt{Lab}' = (\texttt{Lab}'_{\mathcal{A}}, \texttt{Lab}'_{\dashrightarrow})$ of F such that $\{(x, y) \in \dashrightarrow \mid (x, y)$ *has explanatory relevance w.r.t.* $\texttt{Lab}'\} \supsetneq \{(x, y) \in \dashrightarrow \mid (x, y)$ *has explanatory relevance w.r.t.* $\texttt{Lab}\}$.

EC-labeling

Insightful labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF and $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$ a labeling of F. We say that $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$ is an *insightful labeling* of F iff $\mathtt{Lab}$ is an satisfactory labeling of F and there is no satisfactory labeling $\mathtt{Lab}' = (\mathtt{Lab}'_{\mathcal{A}}, \mathtt{Lab}'_{\dashrightarrow})$ of F such that $\{(x, y) \in \dashrightarrow \mid (x, y)$ *has explanatory relevance w.r.t.* $\mathtt{Lab}'\} \supsetneq \{(x, y) \in \dashrightarrow \mid (x, y)$ *has explanatory relevance w.r.t.* $\mathtt{Lab}\}$.

EC-labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF and $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$ a labeling of F. We say that $\mathtt{Lab} = (\mathtt{Lab}_{\mathcal{A}}, \mathtt{Lab}_{\dashrightarrow})$ is an *explanatory core labeling (EC-labeling)* of F iff $\mathtt{Lab}$ is an insightful labeling of F and there is no insightful labeling $\mathtt{Lab}' = (\mathtt{Lab}'_{\mathcal{A}}, \mathtt{Lab}'_{\dashrightarrow})$ of F such that $\{b \in \mathcal{A} \mid \mathtt{Lab}'_{\mathcal{A}}(b) = \mathtt{in}\} \subsetneq \{b \in \mathcal{A} \mid \mathtt{Lab}_{\mathcal{A}}(b) = \mathtt{in}\}$.

Correspondence

AC-labeling correspondence

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF. If $\texttt{Lab}$ is an AC-labeling of F, then $\texttt{Lab2Ext}(\texttt{Lab})$ is an AC-extension of F. Furthermore, if E is an AC-extension of F, then $\texttt{Ext2Lab}(E)$ is an AC-labeling of F.

Correspondence

AC-labeling correspondence

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF. If $\mathtt{Lab}$ is an AC-labeling of F, then $\mathtt{Lab2Ext}(\mathtt{Lab})$ is an AC-extension of F. Furthermore, if E is an AC-extension of F, then $\mathtt{Ext2Lab}(E)$ is an AC-labeling of F.

EC-labeling correspondence

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim \rangle$ be an EAF. If $\mathtt{Lab}$ is an EC-labeling of F, then $\mathtt{Lab2Ext}(\mathtt{Lab})$ is an EC-extension of F. Furthermore, if E is an EC-extension of F, then $\mathtt{Ext2Lab}(E)$ is an EC-labeling of F.

Outline

Extended Explanatory Argumentation Frameworks

EEAF

An *extended explanatory argumentation framework* (EEAF) is a tuple $\langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$, where:

- $\mathcal{A}$ is a set of arguments;
- $\mathcal{X}$ is a set of explananda;
- $\rightarrow$ is a relation of attack;
- $\dashrightarrow$ is a relation of explanation;
- $\sim$ is a relation of incompatibility;
- $\Rightarrow_d$ is a relation of deductive support;
- $\Rightarrow_n$ is a relation of necessary support.

Nature of enriched relations

Incompatibility

An incompatibility is a non-empty set of elements from the EEAF.

Nature of enriched relations

Incompatibility

An incompatibility is a non-empty set of elements from the EEAF.

Other relations (deductive support inverted)

- Sources: $\mathbb{P}(\mathcal{A} \cup \rightarrow \cup \dashrightarrow \cup \sim \cup \Rightarrow_d \cup \Rightarrow_n)$
- Targets: $\mathbb{P}(\mathcal{A} \cup \rightarrow \cup \dashrightarrow \cup \sim \cup \Rightarrow_d \cup \Rightarrow_n) \setminus \emptyset$
- Tag: $\{att, expl, dsup, nsup\}$

Higher order set attacks

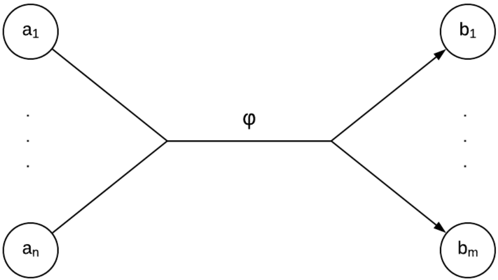
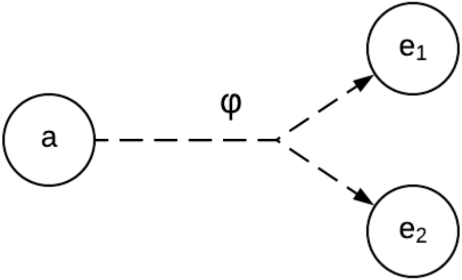


Figure: Set attacking a set via an attack φ.

If all a_is are `in`, then one of the b_ks is `out`.

Disjunctive explanation - Example

A scientific theory based on disjunction of two incompatible assumptions. Potentially explains two explananda, but not both.



Higher order set explanations

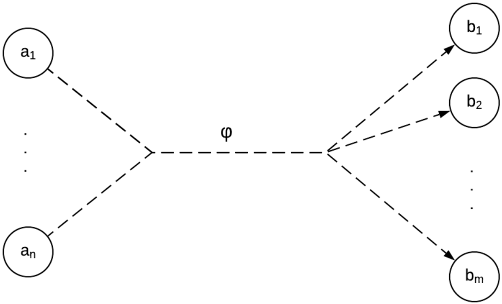


Figure: General case of explanation by a set of elements and of a set of elements.

If all a_is are `in`, then one of the b_ks is explained. Label (φ, b_k) with *expl* and (φ, b_l) with `nexp` for $l \neq k$.

Joint Disjunctive Deductive Support - Example

Professor has 2 PhD students, each with an accepted paper at a conference. There is only budget for one of them to go.

- *a*: The remaining travel budget is Y;
- *b*: Going to the conference costs Y;
- *c*: Student 1 cannot go to the conference;
- *d*: Student 2 cannot go to the conference.

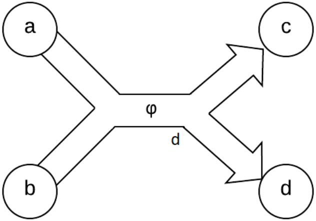


Figure: Example of a set of two arguments deductively supporting another set of two arguments.

Higher order set deductive support

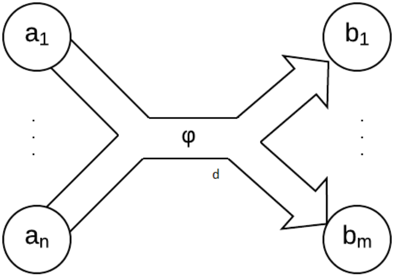


Figure: General case of deductive support by a set of elements and of a set of elements.

If every source is `in`, at least one target is `in`.
I.e. if no target is `in`, at least one source is not `in`.

Higher order set necessary support

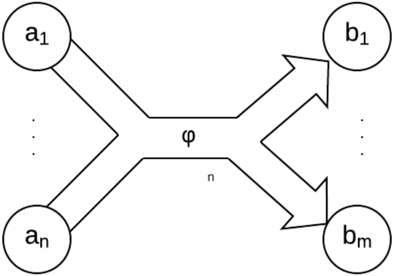


Figure: General case of necessary support by a set of elements and of a set of elements.

Unless at least one element of the source is `in`, we cannot accept every element of the target.
I.e. if no source is `in`, at least one target is not `in`.

Multi-stage label legality

Legal label w.r.t. one attack

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$ be an EEAF, let $\text{Lab} = (\text{Lab}_{NonEx}, \text{Lab}_{PES})$ be a labeling of F. Let b be an element of F such that there is an attack $\varphi \in \rightarrow$ with $b \in \text{trg}(\varphi)$. We say that

- b is *legally* `in` *w.r.t.* `Lab` *and attack* φ iff some element of $\{\varphi\} \cup \text{src}(\varphi) \cup (\text{trg}(\varphi) \setminus \{b\})$ has the acceptance label `out` w.r.t. Lab.
- b is *legally* `out` *w.r.t.* `Lab` *and attack* φ iff every element of $\{\varphi\} \cup \text{src}(\varphi) \cup (\text{trg}(\varphi) \setminus \{b\})$ has the acceptance label `in` w.r.t. Lab.

Multi-stage label legality

Legal label w.r.t. one attack

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$ be an EEAF, let $\text{Lab} = (\text{Lab}_{NonEx}, \text{Lab}_{PES})$ be a labeling of F. Let b be an element of F such that there is an attack $\varphi \in \rightarrow$ with $b \in \text{trg}(\varphi)$. We say that

- b is *legally* `in` *w.r.t. Lab and attack* φ iff some element of $\{\varphi\} \cup \text{src}(\varphi) \cup (\text{trg}(\varphi) \setminus \{b\})$ has the acceptance label `out` w.r.t. Lab.
- b is *legally* `out` *w.r.t. Lab and attack* φ iff every element of $\{\varphi\} \cup \text{src}(\varphi) \cup (\text{trg}(\varphi) \setminus \{b\})$ has the acceptance label `in` w.r.t. Lab.

Legal label w.r.t. attacks

- b is *legally* `in` *w.r.t. Lab and attacks* iff for every attack $\varphi \in \rightarrow$ with $b \in \text{trg}(\varphi)$, b is legally `in` w.r.t. Lab and φ.
- b is *legally* `out` *w.r.t. Lab and attacks* iff for some attack $\varphi \in \rightarrow$ with $b \in \text{trg}(\varphi)$, b is legally `out` w.r.t. Lab and φ.

Aggregating the label legality

Preconditions

$$\text{pre}(x) := \begin{cases} \emptyset & \text{if } x \notin \rightarrow \cup \dashrightarrow \\ \text{src}(x) & \text{if } x \in \rightarrow \cup \dashrightarrow \end{cases}$$

Aggregating the label legality

Preconditions

$$\text{pre}(x) := \begin{cases} \emptyset & \text{if } x \notin \rightarrow \cup \dashrightarrow \\ \text{src}(x) & \text{if } x \in \rightarrow \cup \dashrightarrow \end{cases}$$

Legal label

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$ be an EEAF,
$\texttt{Lab} = (\texttt{Lab}_{NonEx}, \texttt{Lab}_{PES})$ be a labeling of F, and b be an element of F.

- b is *legally* `in` *w.r.t.* `Lab` iff b is legally `in` w.r.t. `Lab` and attacks, incompatibilities, deductive supports and necessary supports and every element of pre(b) has acceptance label `in` w.r.t. `Lab`.
- b is *legally* `out` *w.r.t.* `Lab` iff b is legally `out` w.r.t. `Lab` and either attacks, deductive supports or necessary supports, or some element of pre(b) has acceptance label `out` w.r.t. `Lab`.
- b is *legally* `undec` *w.r.t.* `Lab` iff b is neither legally `in` nor legally `out` w.r.t. `Lab`.

AC-labelings in EEAFs

Define admissibility and completeness as in EAFs based on legality of the labels.

AC-labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$ be an EEAF and $\mathtt{Lab} = (\mathtt{Lab}_{NonEx}, \mathtt{Lab}_{PES})$ a labeling of F. We say that $\mathtt{Lab}$ is an *argumentative core labeling (AC-labeling)* of F iff $\mathtt{Lab}$ is a complete labeling of F and there is no complete labeling $\mathtt{Lab}' = (\mathtt{Lab}'_{NonEx}, \mathtt{Lab}'_{PES})$ of F such that $\{b \in NonEx(F) \mid \mathtt{Lab}'_{NonEx}(b) = \mathtt{in}\} \supsetneq \{b \in NonEx(F) \mid \mathtt{Lab}_{NonEx}(b) = \mathtt{in}\}$ or $\{e \in \mathcal{X} \mid \text{for some } b \in NonEx(F), \mathtt{Lab}'_{PES}((b, e)) = \mathtt{exp}\} \supsetneq \{e \in \mathcal{X} \mid \text{for some } b \in NonEx(F), \mathtt{Lab}_{PES}((b, e)) = \mathtt{exp}\}$.

EC-labelings in EEAFs

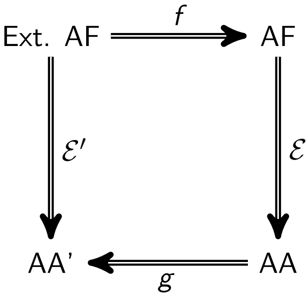
Define explanatory relevance, satisfactory and insightful labelings as in EAFs based on admissibility, explanatory relevance and power.

EC-labeling

Let $F = \langle \mathcal{A}, \mathcal{X}, \rightarrow, \dashrightarrow, \sim, \Rightarrow_d, \Rightarrow_n \rangle$ be an EEAF and $\mathtt{Lab} = (\mathtt{Lab}_{NonEx}, \mathtt{Lab}_{PES})$ a labeling of F. We say that $\mathtt{Lab} = (\mathtt{Lab}_{NonEx}, \mathtt{Lab}_{PES})$ is an *explanatory core labeling (EC-labeling)* of F iff $\mathtt{Lab}$ is an insightful labeling of F and there is no insightful labeling $\mathtt{Lab}' = (\mathtt{Lab}'_{NonEx}, \mathtt{Lab}'_{PES})$ of F such that $\{b \in NonEx(F) \mid \mathtt{Lab}'_{NonEx}(b) = \mathtt{in}\} \subsetneq \{b \in NonEx(F) \mid \mathtt{Lab}_{NonEx}(b) = \mathtt{in}\}$.

Outline

The meta-argumentation methodology[2]



- f: Flattening function
- g: Unflattening function
- $\mathcal{E}$, $\mathcal{E}'$: Acceptance functions

[2] As described in S. Villata's PhD thesis, 2010

Flattening attacks

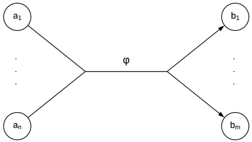


Figure: Set attacking a set via an attack φ.

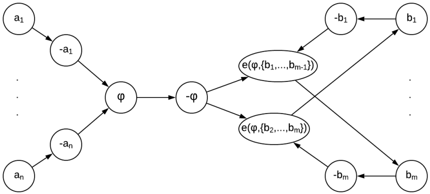


Figure: Flattened attack from the above Figure.

Flattening methodology

- Flatten each relation of the framework locally and uniformly
- Aggregate the flattenings into a large EAF
- Evaluate this EAF
- Translate back to acceptability at the EEAF level

Outline

Conclusion

- Labeling semantics for EAFs
- Aggregation of multiple AF enrichments: EEAFs
- Labeling semantics for EEAFs
- Correspondence with semantics through flattening

Conclusion

- Labeling semantics for EAFs
- Aggregation of multiple AF enrichments: EEAFs
- Labeling semantics for EEAFs
- Correspondence with semantics through flattening

Future work:

- Extend equivalence to other semantics
- Examine benefits of aggregation in other areas

Conclusion

- Labeling semantics for EAFs
- Aggregation of multiple AF enrichments: EEAFs
- Labeling semantics for EEAFs
- Correspondence with semantics through flattening

Future work:

- Extend equivalence to other semantics
- Examine benefits of aggregation in other areas

Thank you!