A Lesson in Default Reasoning

What to agree upon

Emil Weydert

ICR, University of Luxembourg

Agree to Disagree WS - Luxembourg, Nov 10

Our topic

3 questions: given a Tarskian base logic $\mathcal{L} = (L, \vdash_0)$

1. How to interpret defaults?

2. How to plausibly and defeasibly reason with defaults?

3. How to monotonically reason about defaults?

Some answers ...

... to be be aware of (even if one does not agree)!

Defaults

Defaults: different incarnations, realizations

exception-tolerant plausible/normal implications,

defeasible inference tickets, defeasible rules, ...

See as: contingent object-level conditional information

Here: propositional, flat, epistemic/ontic interpretation

Default language: $L(\rightsquigarrow) = \{\varphi \rightsquigarrow \psi \mid \varphi, \psi \in L\}$

$\varphi \rightsquigarrow \psi$: "*if* φ *then by default/defeasibly implies* ψ"

Graded version: $L(\rightsquigarrow_r)_{r \in [0,\infty]_{rat}}$ (countable)

$\twoheadrightarrow = \rightsquigarrow_\infty$: strict/necessary implication

To clarify: Logic, Semantics, Inferential role

Reasoning with defaults

Default inference: *Plausible reasoning using defaults*

Basic finitary inference tasks:

Fact base $\Sigma \subseteq L$, default base $\Delta \subseteq L(\rightsquigarrow, \twoheadrightarrow)$, $\psi \in L$

- *Monotonic inference*: $\Sigma \cup \Delta \vdash \psi$ (or $\Sigma \vdash_\Delta \psi$)
- *Nonmonotonic inference*: $\Sigma \cup \Delta \mid\!\sim \psi$ (or $\Sigma \mid\!\sim_\Delta \psi$)

where $\vdash_0 \subset \vdash \subset \mid\!\sim \subset 2_{fin}^{L_{fin} \cup L(\rightsquigarrow, \twoheadrightarrow)} \times L$

To find: instances + reasonable principles (for $\mid\!\sim$)

Not to confuse: (may fail to imply each other)

- *Object-level defaults*: $\varphi \rightsquigarrow \psi \in \Delta$
- *Meta-level inferential relationships*: $\varphi \mid\!\sim_\Delta \psi$

Examples

Default inference types: roughly two categories

- *Consistency-based* : e.g. Reiter's DL, Argumentation
- *Plausibility-semantic-based* : e.g. Pearl's System Z

Defeasible specificity (DSP): System Z, not RDL, P

- $\{p, p \rightsquigarrow q, q \rightsquigarrow r\} \mid\!\sim r$ (p, q, r logically independent)

- $\{p, p \rightsquigarrow q, q \rightsquigarrow r, p \rightsquigarrow \neg r\} \mid\!\sim \neg r$, but $\not\mid\!\sim r$

Exceptional inheritance (EI): RDL, not System Z

- $\{p, \neg q, p \rightsquigarrow q, p \rightsquigarrow r \mid\!\sim r\}$ (no drowning)

Research in the 90s: seeking the best of both worlds

Proposals $\rightarrow$ Challenges $\rightarrow$ Proposals $\rightarrow$ ... + Principles

Nonmonotonic inference principles for L

Defeasible Modus Ponens (DMP): $\{\varphi\} \cup \{\varphi \rightsquigarrow \psi\} \mid\sim \psi$, but $\{\varphi, \neg\psi\} \cup \{\varphi \rightsquigarrow \psi\} \mid\not\sim \psi$ (φ consistent with $\neg\psi, \psi$)

Preferentiality postulates for $\mid\sim_\Delta$: (Gabbay 85, KLM 90)

- *Supraclassicality* (**SC**): $\vdash_0 \subseteq \mid\sim_\Delta$

- *Left logical equivalence* (**LLE**): $\Sigma' \mid\sim_\Delta \psi$ *if* $\Sigma \dashv\vdash_0 \Sigma', \Sigma \mid\sim_\Delta \psi$

- *Right weakening* (**RW**): $\Sigma \mid\sim_\Delta \psi'$ *if* $\Sigma \mid\sim_\Delta \psi$ and $\{\psi\} \vdash_0 \psi'$

- *Right conjunction* (**AND**): $\Sigma \mid\sim_\Delta \psi \wedge \psi'$ *if* $\Sigma \mid\sim_\Delta \psi, \Sigma \mid\sim_\Delta \psi'$

- *Cautious monotony* (**CM**) + *Cautious transitivity* (**CUT**)

If $\Sigma \mid\sim_\Delta \varphi$, then $\Sigma \mid\sim_\Delta \psi$ *iff* $\Sigma \cup \{\varphi\} \mid\sim_\Delta \psi$

- *Left disjunction* (**OR**): $\{\varphi \vee \varphi'\} \mid\sim_\Delta \psi$ *if* $\{\varphi\} \mid\sim_\Delta \psi, \{\varphi'\} \mid\sim_\Delta \psi$

Consistency Preservation (**CP**): $\Sigma \mid\sim_\Delta \mathbf{F}$ implies $\Sigma \vdash_\Delta \mathbf{F}$

Nonmonotonic inference principles for $L(\leadsto)$

Some core requirements (Weydert 98, 03)

Irrelevance (IRR):

If $\Sigma \cup \Delta \cup \{\psi\}$ and $\Sigma' \cup \Delta' \not\vdash \mathbf{F}$ have disjoint vocabularies, then

- $\Sigma \cup \Delta \mid\!\sim \psi$ iff $\Sigma \cup \Sigma' \cup \Delta \cup \Delta' \mid\!\sim \psi$

Strong representation independence (RI):

If a translation τ respects all the connectives, then

- $\Sigma \cup \Delta \mid\!\sim \psi$ iff $\tau(\Sigma) \cup \tau(\Delta) \mid\!\sim \tau(\psi)$

LLE for defaults: w.r.t. a conditional logic $(L(\leadsto), \vdash_{def})$

- If $\Sigma \mid\!\sim_{\Delta} \psi$ and $\Delta \dashv\vdash_{def} \Delta'$ then $\Sigma \mid\!\sim_{\Delta'} \psi$

Some disappointments

- **RDL** (Reiter 80): not CM, OR, DSP
- **Z** (Pearl 90): not IRR, EI
- **ME** (GMP 03): not RI
- **LEX** (Lehmann 95): violates ME standards even for simple Δ
- ...

Ranking-measure-based default entailment

Ranking measures: $R : Prop_L \to [0, \infty]$ (Spohn 88, 12)

$R(W) = 0$, $R(\emptyset) = \infty$, $R(A \cup B) = \min_{\leq}\{R(A), R(B)\}$

$R(B|A) = R(A \cap B) - R(A)$ for $R(A) \neq \infty$ ($\to$ disbelief)

Semantics: $R \models_{rk}^{\geq 1} \varphi \rightsquigarrow \psi$ iff $R(\varphi \wedge \psi) + 1 \leq R(\varphi \wedge \neg\psi)$

Ranking choice functions: $\mathcal{I} : \Delta \mapsto \mathcal{I}(\Delta) \subseteq Mod_{rk}(\Delta)$

Rkm-based default entailment: $\mid\!\sim^{\mathcal{I}}$ (Wey 96, 98, 03)

- $\Sigma \cup \Delta \mid\!\sim^{\mathcal{I}} \psi$ iff for all $R \in \mathcal{I}(\Delta)$, $R \models_{rk}^{>0} \wedge\Sigma \rightsquigarrow \psi$
- $\Sigma \cup \Delta \vdash_{rk} \psi$ iff $\Delta \vdash_{rk} \wedge\Sigma \wedge \neg\psi \rightsquigarrow \mathbf{F}$

Examples: $\mathcal{I}^P(\Delta) = Mod_{rk}(\Delta)$, $\mathcal{I}^Z(\Delta) = \{Min_{\leq}(Mod_{rk}(\Delta))\}$

Chasing principles

Defeasible modus ponens:

$\{\varphi\} \cup \{\varphi \rightsquigarrow \psi\} \cup \Delta \mid\!\sim^{\mathcal{I}} \psi$ because $\{\varphi \rightsquigarrow \psi\} \cup \Delta \vdash_{rk} \varphi \rightsquigarrow \psi$

$\{\varphi, \neg\psi\} \cup \{\varphi \rightsquigarrow \psi\} \not\mid\!\sim \psi$ for non-degenerate $\mid\!\sim^{\mathcal{I}}$ if $\varphi \wedge \neg\psi \not\vdash \mathbf{F}$

Preferentiality (KLM): true for any $\mid\!\sim^{\mathcal{I}}_{\Delta}$

Idea: focus on ranking constructions (Wey 96), i.e.

$R = \sum_{\delta \in \Delta} r_\delta [\![\varphi_\delta \wedge \neg\psi_\delta]\!]$ $\sim$ adding weights to exception areas

JJ, JZ: Justifiable constructions, also canonical (Wey 98)

$\mathcal{I}^{J}(\Delta) \supset \mathcal{I}^{JJ}(\Delta) \supset \mathcal{I}^{ME}_{=1}(\Delta), \mathcal{I}^{JJR}_{=1}(\Delta), \mathcal{I}^{JZ}_{=1}(\Delta))$

All: DSP, EI, CP, IRR **JX:** RI $\neq$ **J:** $=$ ME on minimal core sets

Logic for defaults

What about: Left logical equivalence LLE_{def} for defaults ?

An inconvenient truth: LLE_{def} holds only for suboptimal $\mid\!\sim^{\mathcal{I}}$ (e.g. violating IRR, EI), like Systems P, Z

EI$_0$: $\{\neg a\} \cup \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b\} \mid\!\sim b$ (for logically independent a, b)

Exceptional inheritance paradox: DMP, SC, EI$_0$, RI, LLE_{def} *are incompatible if* $\vdash_{def}$ *verifies* RW *and* AND

How strong can the conditional logic $(L(\rightsquigarrow), \vdash_{def})$ be without blocking major inferential desiderata?

Which conditional axioms are compatible with which $\mid\!\sim^{\mathcal{I}}$?

Conditional axioms and inference

Weak right weakening: $\varphi \rightsquigarrow \psi \vdash \varphi \rightsquigarrow \psi \vee \neg\varphi$

Weak right conjunction: $\varphi \rightsquigarrow \psi \vdash \varphi \rightsquigarrow \varphi \wedge \psi$

Some observations: (valid/fails means acceptable/inacceptable)

REFL, LLE, WRW, WRC valid for JJ, JJR, JZ, ME

RW valid for ME - fails for JJ, JJR, JZ - repairable

AND, OR fail for all

CM valid for ME, fails for JJ, JJR, JZ - repairable

CUT fails for ME - valid for JJ, JJR, JZ

RDL: validates REFL, LLE, AND, CM

RDL: violates WRW, CUT, OR

System Z: validates all - but violates of course IRR

Main lessons

- To distinguish: defaults $\neq$ inferential relationships
- Ranking-based default entailment is a powerful concept
- Different, incompatible desiderata, necessary trade-off
- Nice inference notions restrict LLE to weak conditional logics
- More research to be done ...
- More to agree upon ...