# Behavioral Social Choice: What is it? 

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$\checkmark$ National Science Foundation (NSF)

> Behavioral Social choice
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Comaniz

Cambridge University Press (2006)
With B. Grofman, A.A.J. Marley, I. Tsetlin
(+ APA analyses with S. Popov, A. Popova, and others)

## Outline

- Behavioral Social Choice

■ "Impossibility" in Social Choice

- Condorcet Paradox
- Partial Ranking Ballots (STV elections of APA)

■ Model Dependence and Bootstrap

- Empirical Consensus among Consensus Methods

2 Conceptual Distinctions in the Decision Sciences

| Normative <br> Theory |
| :---: |

> Descriptive
> Theory \& Data

Individual
Judgment and
Decision Making
Behavioral Decision Research

Social
Choice

## 2 Conceptual Distinctions in the Decision Sciences

| Normative |
| :---: |
| Theory |

## Individual

Judgment and
Decision Making

Social
Choice


## 2 Conceptual Distinctions in the Decision Sciences



| Individual <br> Judgment <br> and <br> Decision Making |
| :---: |

Social


Choice

## A hypothetical profile

3 candidates (A, B, C)
13 voters

|  |  |  | $\#$ |
| :--- | :--- | :--- | :--- |
| A | B | C | 5 |
| C | B | A | 3 |
| C | A | B | 1 |
| B | C | A | 3 |
| A | C | B | 1 |

## Borda

| 2 | 1 | 0 |  | Borda score |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\#$ | A | B | C |
| A | B | C | 5 | 10 | 5 |  |
| C | B | A | 3 |  | 3 | 6 |
| C | A | B | 1 | 1 |  | 2 |
| B | C | A | 3 |  | 6 | 3 |
| A | C | B | 1 | 2 |  | 1 |

## Borda

| 2 | 1 | 0 |  | Borda score |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\#$ | A | B | C |  |  |  |  |  |
| A | B | C | 5 | 10 | 5 |  |  |  |  |  |  |
| C | B | A | 3 |  | 3 | 6 |  |  |  |  |  |
| C | A | B | 1 | 1 |  | 2 |  |  |  |  |  |
| B | C | A | 3 |  | 6 | 3 |  |  |  |  |  |
| A | C | B | 1 | 2 |  | 1 |  |  |  |  |  |
|  |  |  | Total Borda Score |  |  |  |  |  | 13 | 14 | 12 |

Plurality

| 1 |  |  |  |  | Plurality score |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\#$ | A | B | C |  |
| A | B | C | 5 | 5 |  |  |  |
| C | B | A | 3 |  |  | 3 |  |
| C | A | B | 1 |  |  | 1 |  |
| B | C | A | 3 |  | 3 |  |  |
| A | C | B | 1 | 1 |  |  |  |

## Plurality

| 1 |  |  |  | Plurality score |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\#$ | A | B | C |
| A | B | C | 5 | 5 |  |  |
| C | B | A | 3 |  |  | 3 |
| C | A | B | 1 |  |  | 1 |
| B | C | A | 3 |  | 3 |  |
| A | C | B | 1 | 1 |  |  |
|  |  |  | Total Plurality Score | 6 | 3 | 4 |

## Single Transferable Vote (Alternative Vote)

|  |  |  |  |  | sc |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | \# | A | B | C |
| A |  | C | 5 | 5 |  |  |
| C |  | A | 3 |  |  | 3 |
| C | A |  | 1 |  |  | 1 |
|  | C | A | 3 |  |  | 3 |
| A | C |  | 1 | 1 |  |  |

## Single Transferable Vote (Alternative Vote)

|  |  |  |  |  | score |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | \# | A | B C |
| A |  | c | 5 | 5 |  |
| C |  | A | 3 |  | 3 |
| C | A |  | 1 |  | 1 |
|  | C | A | 3 |  | 3 |
| A | C |  | 1 | 1 |  |
|  |  |  | Total | 6 | 7 |

## For now:



## Condorcet (Majority)

|  |  |  | $\#$ | $A>B$ | $B>A$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C | 5 | 5 |  |
| C | B | A | 3 |  | 3 |
| C | A | B | 1 | 1 |  |
| B | C | A | 3 |  | 3 |
| A | C | B | 1 | 1 |  |

## Condorcet (Majority)

|  |  |  | $\#$ | $A>C$ | C>A |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C | 5 | 5 |  |
| C | B | A | 3 |  | 3 |
| C | A | B | 1 |  | 1 |
| B | C | A | 3 |  | 3 |
| A | C | B | 1 | 1 |  |

## Condorcet (Majority)

|  |  |  | $\#$ | $B>C$ | C>B |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C | 5 | 5 |  |  |
| C | B | A | 3 |  | 3 |  |
| C | A | B | 1 |  | 1 |  |
| B | C | A | 3 | 3 |  |  |
| A | C | B | 1 |  |  | 1 |

## Are Consensus Methods Irreconcilable?

| Voting <br> Rule | Borda | Plurality | STV | Condorcet |
| :---: | :---: | :---: | :---: | :---: |
| Winner | B | A | C | cycle |



## State of the Art: Shepsle et al. 1997

Probability of a Cycle: $\operatorname{Pr}(\mathrm{m}, \mathrm{n})$
Based on Sampling from a Uniform Distribution on Linear Orders ("Impartial Culture")*

|  | number of voters (n) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| number of <br> alternatives <br> $(\mathbf{m})$ | 3 | 5 | 7 | 9 | 11 | limit |
| 3 | .056 | .069 | .075 | .078 | .080 | .088 |
| 4 | .111 | .139 | .150 | .156 | .160 | .176 |
| 5 | .160 | .200 | .215 |  |  | .251 |
| 6 | .202 |  |  |  |  | .315 |
|  |  |  |  |  |  |  |
| limit | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ |

*Source: Riker (1982: 122) as reproduced in Shepsle and Bonchek (1997: Table 4.1, 54)

## State of the Art: Shepsle et al. 1997

| Probability of a Cycle: $\operatorname{Pr}(m, n)$ <br> Based on Sampling from a Uniform Distribution on Linear Orders ("Impartial Culture")* |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | number of voters (n)   <br> 3 5  |  |  |  |  |  |
| number of alternatives (m) | 3 | 5 | 7 | 9 | 11 | limit |
| 3 | . 056 | . 069 | . 075 | . 078 | . 080 | . 088 |
| 4 | . 111 | . 139 | . 150 | . 156 | . 160 | . 176 |
| 5 | . 160 | . 200 | . 215 |  |  | 251 |
| 6 | . 202 |  |  |  |  | 315 |
|  |  |  |  |  |  |  |
| limit | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ |

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| 6 | .202 |  |  |  |  | .315 |
|  |  |  |  |  |  |  |
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|  | Probability of a Cycle: Pr(m, n$)$ <br> Based on Sampling from a Uniform Distribution on Linear Orders <br> ("Impartial Culture")* |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

*Source: Riker (1982: 122) as reproduced in Shepsle and Bonchek (1997: Table 4.1, 54)

## Shepsle \& Bonchek (1997)

"In general, then, we cannot rely on the method of majority rule to produce a coherent sense of what the group 'wants', especially if there are no institutional mechanisms for keeping participation restricted (thereby keeping $\boldsymbol{n}$ small) or weeding out some of the alternatives (thereby keeping $m$ small)."

## \$1,000,000 Question:

Where is the empirical evidence
for voting paradoxes in practice?
Oops....
For instance, hardly any evidence that majority cycles have ever occurred among serious contenders of major elections.

Actually, evidence circumstantial at best.

## Where is the evidence for cycles?

## Majority Winner

- Candidate who is ranked ahead of any other candidate by more than $50 \%$
- Candidate who beats any other candidate in pairwise competition

Plurality: Choose one
SNTV \& Limited Vote: Choose k many Approval Voting: Choose any subset STV (Hare), AV (RIV): Rank top k many
Cumulative Voting: Give $m$ pts to $k$ many
Survey Data: Thermometer, Likert Scales
Data are incomplete!!

## A General Concept of Majority Rule

| Linear Orders | "complete rankings" |
| :--- | :--- |
| Weak Orders | "rankings with possible ties" |
| Semiorders | "rankings with (fixed) threshold" |
| Interval Orders | "rankings with (variable) threshold" |
| Partial Orders | asymmetric, transitive |

Asymmetric Binary Relations



# Variable Preferences: <br> Probability Distribution on Binary Relations 

## Variable Utilities:

Jointly Distributed Family of
Utility Random Variables
(Random Utilities)
(parametric or nonparametric)

## Random Utility Representations

## Linear Orders Weak Orders

$$
P(B)=P\left(\begin{array}{c}
\mathbf{U}_{i}>\mathbf{U}_{j} \mid(i, j) \in B \\
\text { and } \\
\mathbf{U}_{i} \leq \mathbf{U}_{j} \mid(i, j) \notin B
\end{array}\right)
$$

$$
\begin{aligned}
& \text { With } P\left(\mathbf{U}_{i}=\mathbf{U}_{j}\right)=0 \\
& \quad(i \neq j)
\end{aligned}
$$

(Block and Marschak, 1960, chapter)

## Random Utility Representations

Semiorders Interval Orders

$$
P(B)=P\left(\begin{array}{c}
\mathbf{L}_{i}>\mathbf{U}_{j} \mid(i, j) \in B \\
\text { and } \\
\mathbf{L}_{i} \leq \mathbf{U}_{j} \mid(i, j) \notin B
\end{array}\right)
$$

$$
\begin{gathered}
\text { With } \mathbf{U}_{i}(\omega)=\mathbf{L}_{i}(\omega)+\varepsilon \\
\forall \omega
\end{gathered}
$$

## A General Definition of Majority Rule

Given a probability distribution

$$
\begin{aligned}
P: B & \rightarrow[0,1] \\
B & \mapsto P(B)
\end{aligned}
$$

on any set $B$ of binary relations,
$a$ is strictly majority preferred to $b$
if and only if

$$
\sum_{(a, b) \in B} P(B)>\sum_{(b, a) \in B^{\prime}} P\left(B^{\prime}\right)
$$

## A General Definition of Majority Rule

Given a probability distribution
$P: B \rightarrow[0,1]$
$B \mapsto P(B)$
on any set $B$ of binary relations,
$a$ is strictly majority preferred to $b$
if and only if
$\sum_{(a, b) \in B} P(B)>\sum_{(b, a) \in B^{\prime}} P\left(B^{\prime}\right)$

For Utility Functions or Random Utility Models choose a Random Utility Representation and obtain a consistent Definition

## \$1,000,000 Question:

Where is the empirical evidence for voting paradoxes in practice?

Let's analyze National Survey Data! 1968, 1980, 1992, 1996 ANES

Feeling Thermometer Ratings translated into
Weak Orders or Semiorders




## ANES Strict Majority <br> Social Welfare Orders

| Year | Threshold | SWO <br> Nixon <br> Humphrey <br> Wallace |
| :---: | :---: | :---: |

# ANES Strict Majority Social Welfare Orders 

| Year | Threshold | SWO <br> Clinton <br> Bush <br> Perot |
| :---: | :---: | :---: |

## However: <br> There is no Theory-Free Majority Preference Relation

## ANES Strict Majority Social Welfare Orders

| Year | Threshold | SWO <br> Carter <br> Reagan <br> Anderson |
| :---: | :---: | :---: |
| 1980 | $0, \ldots, 29$ | Reagan <br> Carter <br> Anderson |

## ANES Strict Majority <br> Social Welfare Orders

| Year | Threshold | SWO |
| :---: | :---: | :---: |
| 1996 | $0, \ldots, 49$ | Clinton <br> Dole <br> Perot |
|  | $50, \ldots, 99$ | Dole <br> Clinton <br> Perot |

## Preliminary Conclusions:

## Majority Preference Relation is hypothetical is model dependent should be treated in an inference framework may or may not be robust <br> Where are the cycles?

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| 6 | .202 |  |  |  |  | .315 |
|  |  |  |  |  |  |  |
| limit | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ | $\approx 1.00$ |

*Source: Riker (1982: 122) as reproduced in Shepsle and Bonchek (1997: Table 4.1, 54)

## Drawing Random Samples from Realistic Distributions

What happens if we interview 20 randomly drawn voters from the 1996 ANES?

Do they display cyclical majorities?
Do they display the correct majority preference order?









1976 Germany




## American Psychological Association Presidential Elections (Single Transferable Vote)

- Good approximation of political elections

■ 8 Data sets (1998-2005)

- Size +/- 20,000 voters

■ Ballots: Partial/Full Rankings of 5 Candidates

## Two Types of Data

Complete Ranking

- Partial Ranking

B $\quad$ D $\quad$ C $\quad$ E $\quad$ A
B D C

## Three Models of Partial Rankings

- Weak order model
- Unranked candidates are tied at the bottom of the preference
- Partial order model (Zwicker)
- No preference b/w candidates when one or both are unranked



## Three Models of Partial Rankings

■ Size-Independent Linear Order model

- Partial ranking is "beginning" of an unknown linear order
- Linear order "profile" inferred statistically



## Results 1998

| 1998 | Condorcet |
| :--- | :--- |
| WO | CBADE |


| 1998 | Condorcet |  |  | Borda |
| :--- | :--- | :--- | :---: | :---: |
| WO | CBADE | CBADE |  |  |

## Results 1998

| 1998 | Condorcet Borda | Plurality |  |
| :--- | :--- | :--- | :--- |
| WO | CBADE | CBADE | CEABD |

Bootstrapped Confidence bold > 95\%

| $\mathbf{1 9 9 8}$ | Condorcet |  |  |
| :--- | :--- | :--- | :--- |
| Borda | Plurality |  |  |
| WO | CBADE | CBADE | CEABD |
| $\boldsymbol{Z W}$ | CBDAE | CBDAE | CAEBD |



| $\mathbf{1 9 9 8}$ | Condorcet | Borda | Plurality |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{W O}$ | CBADE | CBADE | CEABD |
| $\boldsymbol{Z W}$ | CBDAE | CBDAE | CAEBD |
| $\boldsymbol{S I M}$ | CBDAE | CBDAE | CAEBD |

[^0]Bootstrapped Confidence
Results 2005

| 2005 | Condorcet | Borda | Plurality |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{W O}$ | BADEC | BADEC | $\underline{\text { ABDEC }}$ |
| $\boldsymbol{Z W}$ | BADEC | BADEC | $\underline{\text { ABDEC }}$ |
| $\boldsymbol{S I M}$ | BADEC | BADEC | $\underline{\text { ABDEC }}$ |

Agreement among winners Bootstrapped APA data (Weak Order Model) Confidence bold > 95\%

| Condorcet <br> Borda |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Condorcet <br> Plurality | Borda <br> Plurality | All 3 |  |  |
| 1998 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 1999 | $>.99$ | .79 | .79 | .79 |
| 2000 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2001 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2002 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2003 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2004 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2005 | $>.96$ | .03 | .06 | $<.03$ |

Agreement among winners Bootstrapped APA data (Partial Order Model) Confidence bold $>95 \%$

| Condorcet <br> Borda |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Condorcet <br> Plurality | Borda <br> Plurality | All 3 |  |  |
| 1998 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 1999 | .89 | .86 | .93 | .67 |
| 2000 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2001 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2002 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2003 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2004 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2005 | .89 | .70 | .67 | .63 |

Agreement among winners Bootstrapped APA data (Linear Order Model) $\begin{aligned} & \text { Conidence } \\ & \text { bold }>95 \%\end{aligned}$

| Condorcet <br> Borda |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Condorcet <br> Plurality | Borda <br> Plurality | All 3 |  |  |
| 1998 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 1999 | $>.99$ | 0.79 | 0.79 | 0.75 |
| 2000 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2001 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2002 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2003 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2004 | $>.99$ | $>.99$ | $>.99$ | $>.99$ |
| 2005 | 0.97 | 0.02 | 0.04 | 0.02 |

## Generalized Voting Rules*

Condorcet
Borda
Plurality

- Anti Plurality
- STV
- Coombs
- Plurality Runoff
* Source: Regenwetter \& Rykhlevskaia (2007), Regenwetter et al. (2006)




## Conclusions

- Impossibility theorems and voting paradoxes
- Hypothetical profiles in text books

Behavioral Social Choice:

- Model dependence
- Replicability (bootstrap/statistical confidence)
- Condorcet Paradox appears to be rare
- Empirical consensus among consensus methods (in large scale data)


## Thank you!

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[^0]:    No Cycle

