Dynamics of Argumentation Frameworks with Subargument Relations

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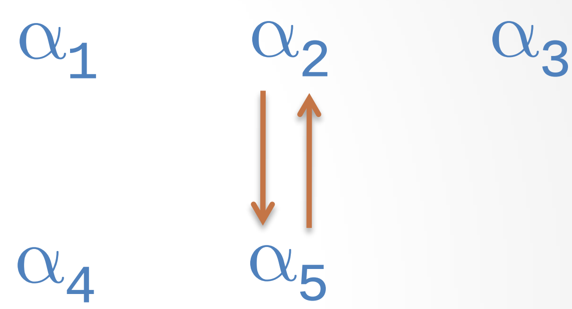
Outline

- Motivation
- Some basic notions
- Argumentation Frameworks with Subargument Relations (AFwSs)
- Layered AFwSs
- Dynamics of an AFwS (Preliminary Considerations)
- Conclusions

→ direct attack

Motivation

{ Quakers are pacifists
Pacifists follow principles of nonviolence
Republicans are not pacifists
Nixon is a Quaker
Nixon is a Republican }



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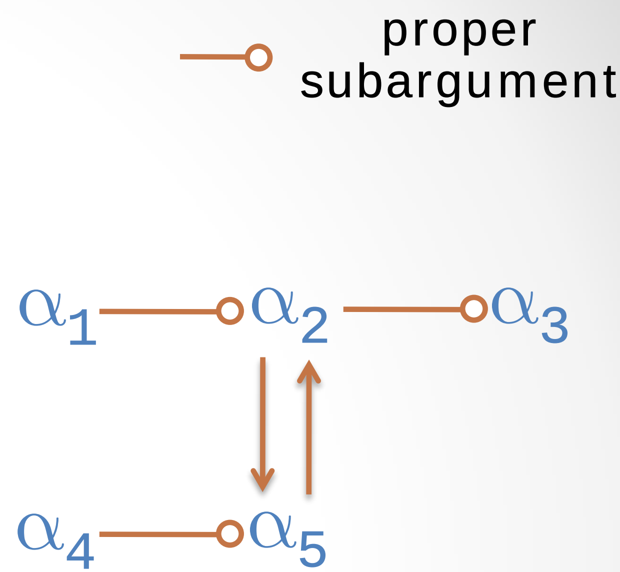

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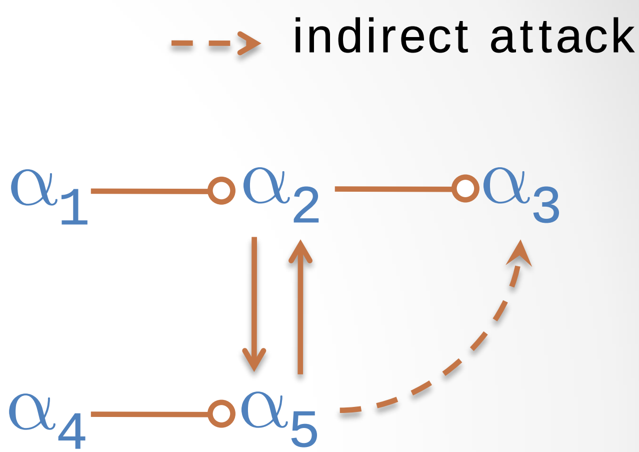

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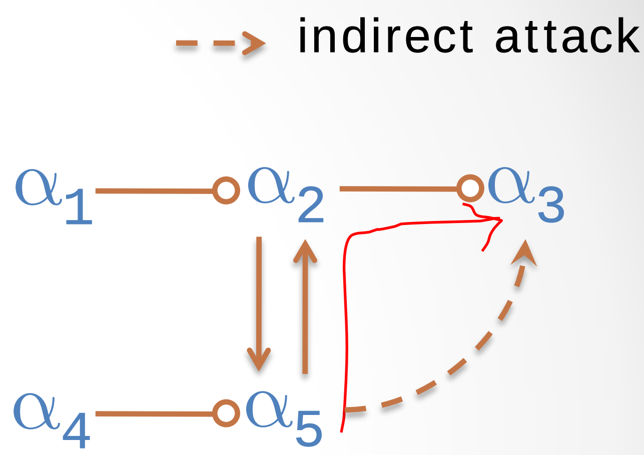

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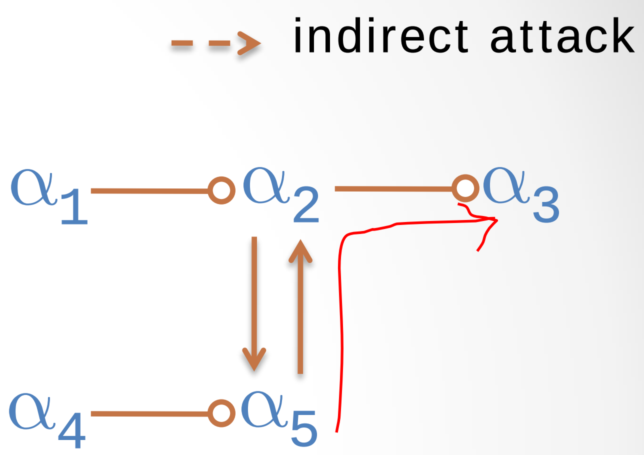

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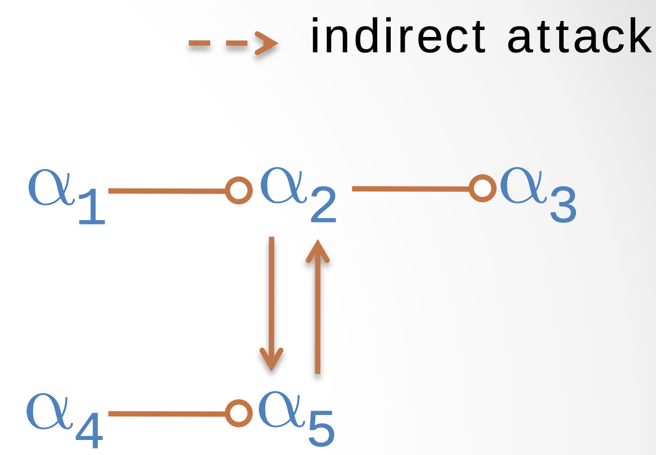

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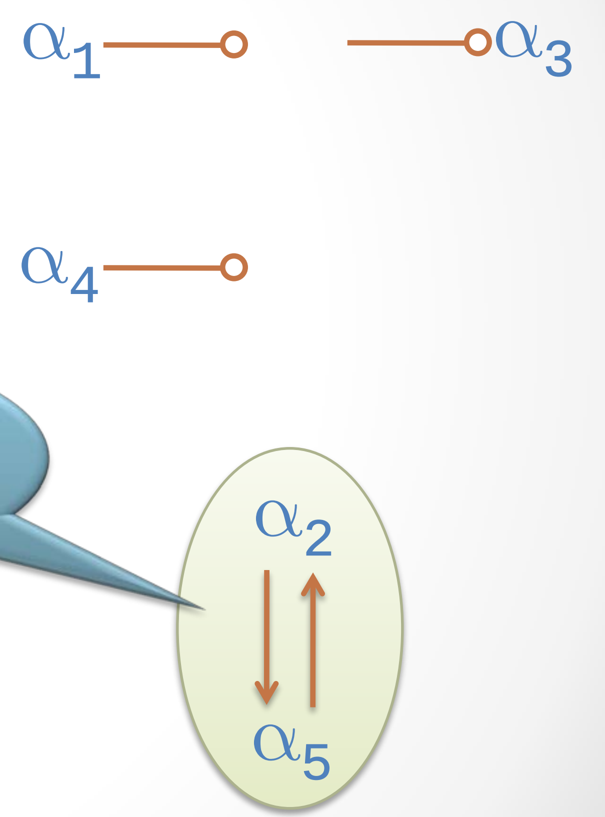

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Conflict-handling arguments

Unattcked set

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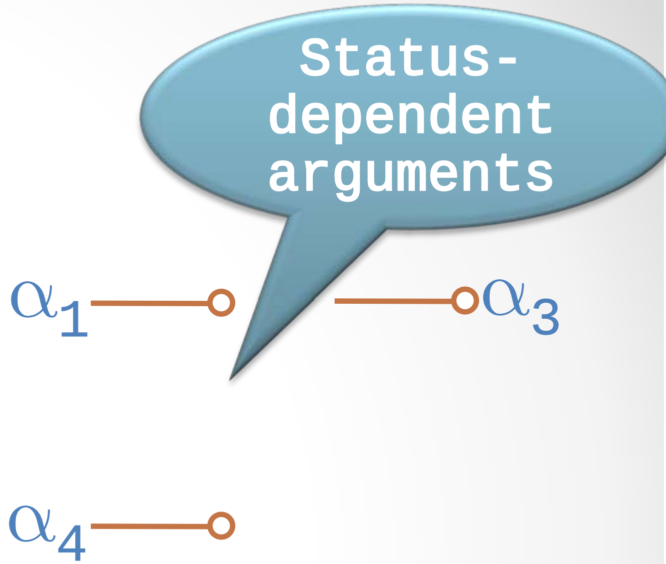

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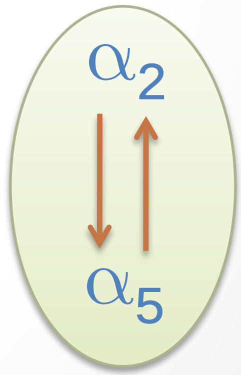

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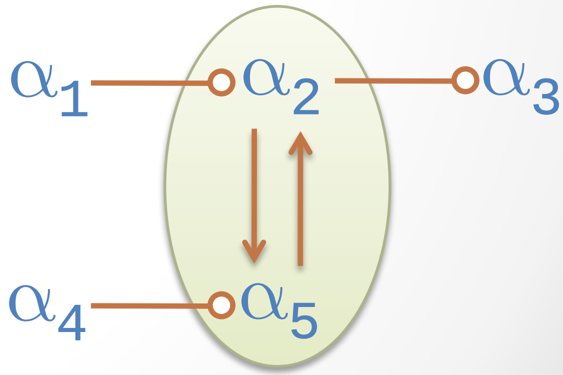

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Motivation

Quakers are pacifists
Pacifists follow principles of nonviolence

Given a status-dependent argument, if all of its proper subarguments are acceptable, then it is acceptable.

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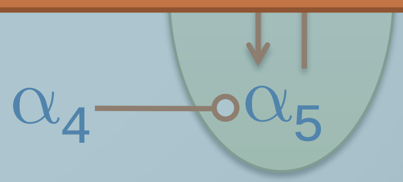


Motivation (Cont.)

- For an AFwS, the status of some structured arguments depends on that of their proper subarguments.
- Not all arguments should be involved in the process of acceptability evaluation of arguments.
- The status evolution of arguments is related not only to the attack relations, but also to the subargument relations.

Motivation (Cont.)

- So, we propose a layered AFwS, such that
  - o the conflicts among arguments are handled centrally;
  - o the status evolution of arguments is characterized not only by the attack relations, but also by the subargument relations.

Some Basic Notions

- Structured arguments
- Subarguments
- Direct attacks and indirect attacks between structured arguments

Structured arguments

- An (abstract) logical language $\mathcal{L}$ (a set)
- A contrariness function $^-$ from $\mathcal{L}$ to $2^{\mathcal{L}}$: if $p \in \overline{q}$, then if $q \notin \overline{p}$ then p is called a contrary of q, otherwise p and q are called contradictory.

--- [Prakken 2012]

- Two kinds of rules
  - o Strict rules of the form $p_1,\ldots,p_n \rightarrow q$ ($n \geq 0$)
  - o Defeasible rules of the form $p_1,\ldots,p_n \Rightarrow q$ ($n \geq 0$)
- A defeasible theory **T**= (**S**,**D**), in which **S** is a set of strict rules, and **D** a set of defeasible rules.

Structured arguments (Cont.)

- With respect to a defeasible theory **T**= (**S**,**D**), a structured argument α is a tuple (H,q), in which
  - o q is the conclusion of α, denoted by Conc(α) =q
  - o H is the premise of α, denoted by Prem(α) =H:

    when $H = \varnothing$,

    there exists an axiom $\rightarrow q$ in **S**,

    or an assumption $\Rightarrow q$ in **D**,

    such that Conc(α) =q;

    when H={α_1, ..., α_n } ($n \geq 1$),

    there exists a strict rule Conc(α_1), ...,Conc(α_n) $\rightarrow q$ in **S**,

    or a defeasible rule Conc(α_1), ..., Conc(α_n) $\Rightarrow q$ in **D**,

    such that Conc(α) =q.

Subarguments

- Let Arg(**T**) be a set of arguments constructed from a defeasible theory **T**=(**S**,**D**), and $\alpha=(H,q)$ an argument in Arg(**T**).
  - The set of subarguments of α is recursively defined as:

    $\mathrm{Sub}(\alpha) =_{def} (\cup_{\beta \in H} \mathrm{Sub}(\beta)) \cup \{\alpha\}$

  - The set of superarguments of α is defined as:

    $\mathrm{Sup}(\alpha) =_{def} \{\beta \in \mathrm{Arg}(\mathbf{T}) \mid \alpha \in \mathrm{Sub}(\beta)\}$

We call $\mathrm{Sub}(\alpha)\backslash\{\alpha\}$ the set of proper subarguments of α, and $\mathrm{Sup}(\alpha)\backslash\{\alpha\}$ the set of proper superarguments of α.

Proposition 1. The (proper) subargument relations are transitive.

Direct Attacks and Indirect Attacks

- Let α and β be structured arguments in Arg(**T**)
  - o α directly attacks β , if and only if
    - (i) Conc(α) and Conc(β) are contradictory, and $\alpha \not\prec \beta$; or
    - (ii) Conc(α) is a contrary of Conc(β); or
    - (iii) Conc(α) is a contrary of β.
  - o α indirectly attacks β, if and only if $\exists\gamma\in$Sub(β)\{β}, such that α directly attacks γ.

  The set of direct (indirect) attacks between the arguments in Arg(**T**) is denoted as *direct* (respectively, *indirect*)

Proposition 2. For all $\alpha,\beta\in$Arg(**T**), if α directly attacks β, then there exists γ, a proper subargument of β, such that α directly attacks γ; if α attacks β (directly or indirectly), then α indirectly attacks all proper superarguments of β.

AFwS

- An argumentation framework with subargument relations (AFwS) is defined as 4-tuple:

  $AFwS =_{def} (\text{Arg}(\mathbf{T}), \textit{direct}, \textit{indirect}, \text{Sub})$

  The corresponding Dung-style AF is

  $AF = (\text{Arg}(\mathbf{T}), \textit{direct} \cup \textit{indirect})$

- Status dependence among the arguments of an AFwS

Theorem 1. Let $AFwS = (\text{Arg}(\mathbf{T}), \textit{direct} \cup \textit{indirect}, \text{Sub})$, and $B \subseteq \text{Arg}(\mathbf{T})$. Given a status-dependent argument $\alpha \in \text{Arg}(\text{T})$, α is accptable w.r.t. *B*, if and only if $\forall \beta \in \text{Sub}(\alpha) \setminus \{\alpha\}$, β is acceptable w.r.t. *B*.

Layered AFwS

- Given $AFwS$ = (Arg(**T**), *direct*, *indirect*, Sub), we may divide Arg(**T**) into three subsets:
  - a set of conflict-handling arguments, denoted A_c;
  - q set of non-trivial status-dependent arguments, denoted A_d ;
  - a set of trivial status-dependent arguments, denoted A_u.

$A_c =_{def} \{\alpha \in \text{Arg}(\mathbf{T}) \mid \exists\beta\in\text{Arg}(\mathbf{T}): (\alpha,\beta) \in \textit{direct}, \text{ or } (\beta,\alpha) \in \textit{direct}\}$

$A_d =_{def} \{\beta \in \text{Arg}(\mathbf{T})\setminus A_c \mid \exists\alpha\in\text{Arg}(\mathbf{T}): (\alpha,\beta) \in \textit{indirect}\}$

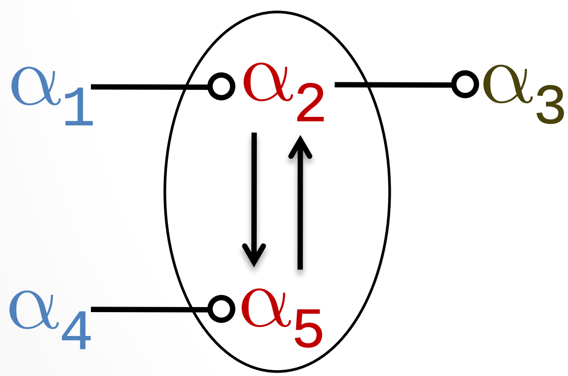
$A_u =_{def} \text{Arg}(\mathbf{T})\setminus (A_c \cup A_d)$

Proposition 3. It holds that: (i) A_c is an unattacked set; (ii) A_d is conflict-free ; (iii) arguments in A_u are unrelated to any attacks; and (iv) A_c , A_d and A_u is a partition of Arg(**T**).

Layered AFwS (Cont.)

- Given $AFwS$ = (Arg(**T**), *direct*, *indirect*, Sub), A_c , A_d and A_u, a layered AFwS (LAFwS) is defined as follows:

$$LAFwS =_{def} (\langle A_c , R_c \rangle, (A_d , A_u), \text{Sub})$$



$A_c = \{\alpha_2, \alpha_5\}$
$A_d = \{\alpha_3\}$
$A_u = \{\alpha_1, \alpha_4\}$
$\text{Sub}(\alpha_1) = \{\alpha_1\}$
$\text{Sub}(\alpha_2) = \{\alpha_1, \alpha_2\}$
$\text{Sub}(\alpha_3) = \{\alpha_1, \alpha_2, \alpha_3\}$
$\text{Sub}(\alpha_4) = \{\alpha_4\}$
$\text{Sub}(\alpha_5) = \{\alpha_4, \alpha_5\}$

Layered AFwS (Cont.)

- The extensions of *LAFwS* $=(\langle A_c, R_c\rangle, (A_d, A_u), \text{Sub})$ is obtained by expanding each extension of $\langle A_c, R_c\rangle$ with the arguments in A_d and A_u.
- The expansion function of *LAFwS* is defined as follows:

$$\pi_{LAFwS}: 2^{A_c \cup A_d} \to 2^{A_c \cup A_d},$$
$$\pi_{LAFwS}(B) = B \cup \{\alpha \in A_d \setminus B \mid (\text{Sub}(\alpha) \setminus \{\alpha\}) \subseteq B \cup A_u\}.$$

Proposition 4. π_{LAFWS} is monotonic (w.r.t set inclusion).

Then, given $B \subseteq A_c \cup A_d$, there exists a unique fixed point of π_{LAFWS}, denoted as B^*.

Theorem 2. If B is admissible, then B^* is admissible.

Layered AFwS (Cont.)

- Let $LAFwS = (\langle A_c, R_c \rangle, (A_d, A_u), \text{Sub})$ be a layered AFwS, and $AF = \langle \text{Arg}(\mathbf{T}), direct \cup indirect \rangle$ the corresponding Dung-style argumentation framework, where $\text{Arg}(\mathbf{T}) = A_c \cup A_d \cup A_u$.
- Given the condition that the number of arguments in Arg(**T**) is finite, and there exists no circular argument in Arg(**T**), we have proved that under complete, ground, preferred and stable semantics, respectively, the following property holds (we are now trying to prove this property under some other semantics):

Theorem 3. $\mathcal{E}_{\mathcal{S}}(AF) = \{E^* \cup A_u \mid E \in \mathcal{E}_{\mathcal{S}}(\langle A_c, R_c \rangle)\}$.

Dynamics of an AFwS
(Preliminary Considerations)

- Add a set of arguments to a layered AFwS
- Remove a set of arguments from a layered AFwS

Adding a set of arguments

- Let P ($P \cap \text{Arg}(\mathbf{T}) = \varnothing$) be a set of arguments to be added to $LAFwS = (\langle A_c, R_c \rangle, (A_d, A_u), \text{Sub})$, along with a set of direct attacks S and sets of subarguments. Then, P can be divided into three subsets P_c, P_d and P_u, such that:

$$P_c = \{\alpha \in P \mid \exists \beta \in \text{Arg}(\mathbf{T}) \cup P_c: (\alpha, \beta) \in S, \text{ or } (\beta, \alpha) \in S\},$$

$$P_d = \{\alpha \in P \setminus P_c \mid \exists \beta \in \text{Sub}(\alpha) \setminus \{\alpha\}: (\beta, \alpha) \in S\},$$

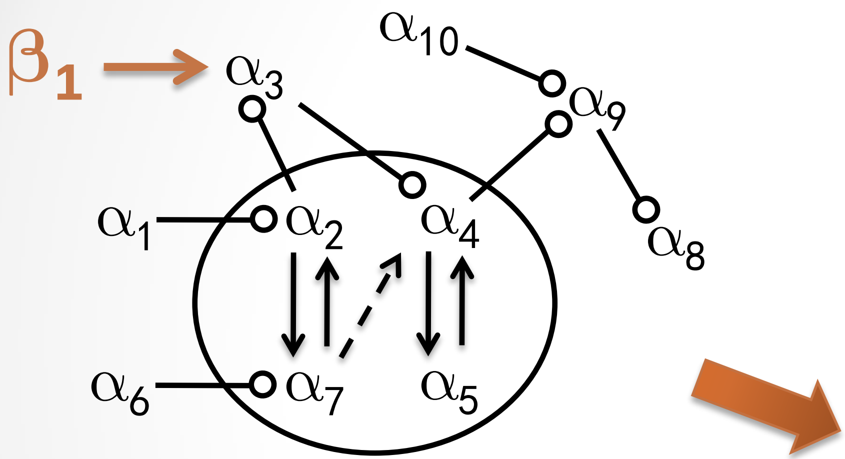
$$P_u = P \setminus (P_c \cup P_d).$$

Adding a set of arguments (Cont.)

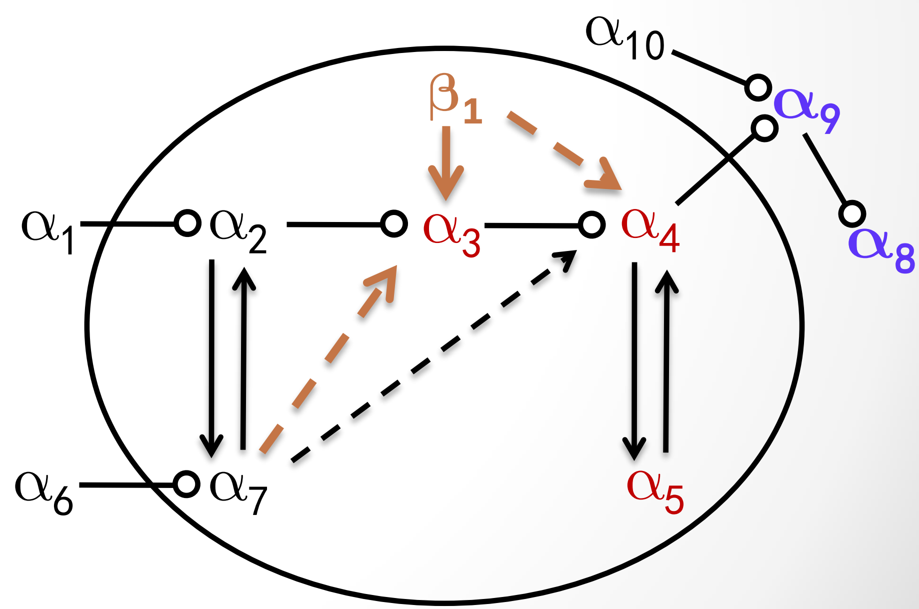
- The dynamics of the AFwS is computed in two steps:
  - First, identify a set of conflict-handling arguments that are affected by P_c (w.r.t. attack relations), denoted as Aff(Arg(**T**), P_c), and compute the status of these arguments by a division-based method.

    [Beishui Liao, et al. Dynamics of Argumentation Systems: A Division-Based Method. Artificial Intelligence, 2011]
  - Second, identify a set of status-dependent arguments that are affected by Aff(Arg(**T**), P_c), P_d and P_u (w.r.t. subargument relations), and compute the status of these arguments by exploiting the expansion function of a layered AFwS.

Adding a set of arguments (Cont.)



The set of affected status-dependent arguments: $\{\alpha_8, \alpha_9\}$

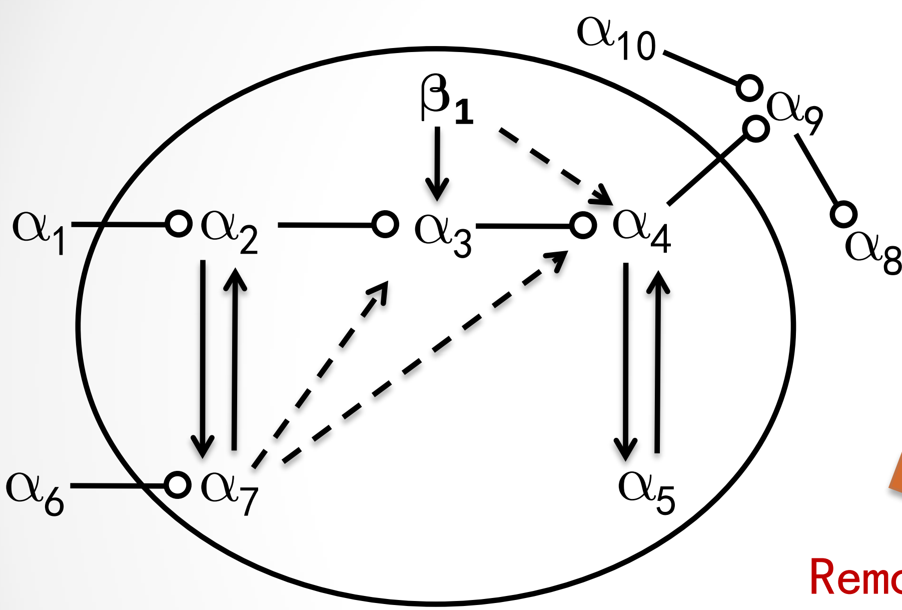


The set of affected conflict-handling arguments: $\{\beta_1, \alpha_3, \alpha_4, \alpha_5\}$

Removing a set of arguments

- Let $Q \subseteq \mathrm{Arg}(\mathbf{T})$ be a set of arguments to be removed from $LAFwS = (\langle A_c, R_c \rangle, (A_d, A_u), \mathrm{Sub})$.
- After removing Q, some arguments in $A_c \backslash Q$ that are no longer related to direct attacks are added to $A_d \backslash Q$ or $A_u \backslash Q$.
- Then, the dynamics of the AFwS can be computed by a method similar to the one in the case of adding a set of arguments.

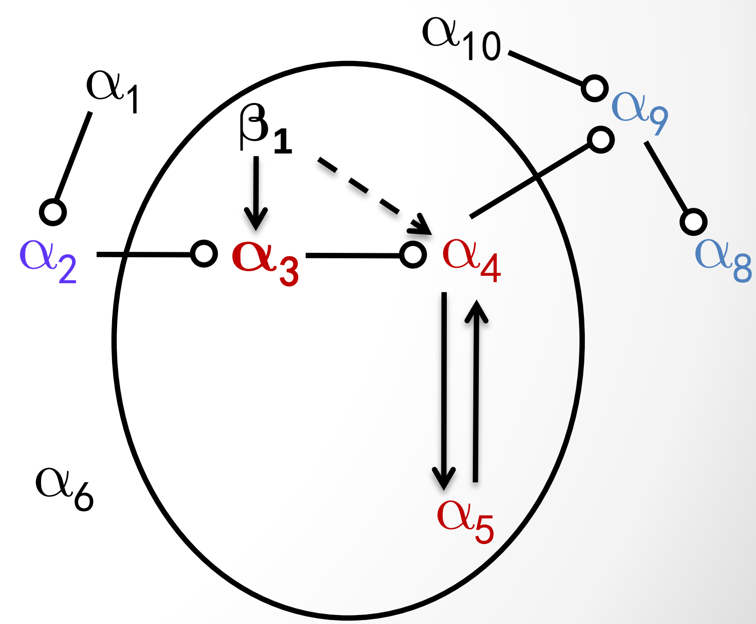
Removing a set of arguments (Cont.)



The set of affected conflict-handling arguments: $\{\alpha_3, \alpha_4, \alpha_5\}$

Remove α_7

The set of affected status-dependent arguments: $\{\alpha_8, \alpha_2, \alpha_9\}$



Conclusion

- We have proposed a layered argumentation framework with subargument relations (LAFwS), and discussed the dynamics of an AFwS.
- The novelty of this work is twofold:
  - o The idea of formulating a theory of argumentation by exploiting the subargument relations
  - o The notion of the expansion function of a LAFwS, which lays a foundation of formulating the semantics, and the dynamics, of an AFwS

Thank you!