Evolutions and Revolutions in Rule Change

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Re-vision of AGM

- The AGM postulates have no more 'rationality' than Church axioms for classical logic. Their justification is solely based on the semantic modeling.
- AGM models are *abstract*; they are (at best) only an output of more complex semantic structures of epistemic states.
- The set of fundamental operations on epistemic states is much richer than contraction-expansion-revision.
- AGM obliterates the distinction between factual and rule change.

The Second Side of the Coin: AGM versus NMR

- NMR theories provide an insight into the nature and functional role of epistemic states.
- Belief formation and belief change occur as a result of combining facts with epistemic states.
- Epistemic states are relatively stable entities; they are *normally not changed* by the facts.

Rules (conditionals) constitute the core of epistemic states.

Facts vs Rules: Inertia Principles

Commonsense inertia for facts

Factual propositions should preserve their previous *truth-values* unless affected by rules(actions).

Factual change is an essentially temporal process.

Commonsense inertia for rules

Rules should preserve *validity* whenever not canceled (refuted) by facts or other rules [The *causal rejection* principle in LP].

Rules are essentially atemporal: accumulation and inheritance; more recent rules are deemed more ‘specific’.

Evolutionary Rule Change

- *Ordinary (evolutionary) rule change = prioritized accumulation of rules.*
- Evolutionary rule change is representable in a static framework of a prioritized set of rules.
- Derogation of a rule arises due to eventual reduction of its applicability range by 'better' rules.

If $A \rightarrow B$ is prior to $A \rightarrow \neg B$, the latter becomes inapplicable.

Preferential Approach

Priority = Preference

- Origins: McCarthy's circumscription:

A normally implies *B* : $(A \rightarrow B) \equiv A \wedge \neg ab \supset B$

- Circumscription policy: minimization of abnormality.
- Interaction of conflicting defaults: aspects of abnormality and prioritized circumscription.

Prioritized rule base

$(\Delta, \rhd)$, where Δ is a set of rules (conditionals), and $\rhd$ is a *priority relation* on Δ. $\beta \rhd \alpha$— α *is prior to* β.

An associated epistemic state $(\mathcal{S}, \prec)$

- $\mathcal{S} = \mathcal{P}(\Delta)$ is the set of *belief states*;
- $\prec$ is a monotonic *preference relation* on $\mathcal{P}(\Delta)$: if $\Gamma \subset \Phi$, then $\Gamma \prec \Phi$.

Accepted rules are obtained by choosing preferred belief states consistent with the facts.

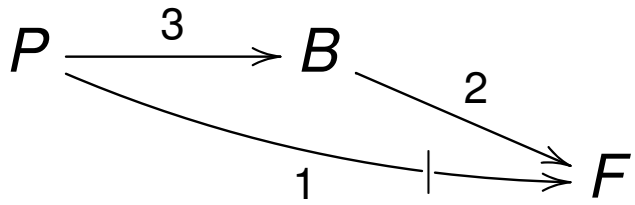
The main problem: a principled way of constructing the preference order $\prec$ from $\rhd$.

Defeasible entailment

Desiderata

Direct Inference If $A \to B \in \Delta$, then $A \supset B$ should hold in all preferred states consistent with A;

Specificity More specific rules should be preferred to less specific ones in cases of conflict.



$\{1, 3\}$ should be preferred to $\{2, 3\}$ for P.

Theories of defeasible entailment: rational closure, system Z, lexicographic inference, conditional entailment, nonmonotonic inheritance, etc. etc.

Conditional entailment (Geffner)

The consensus preference order

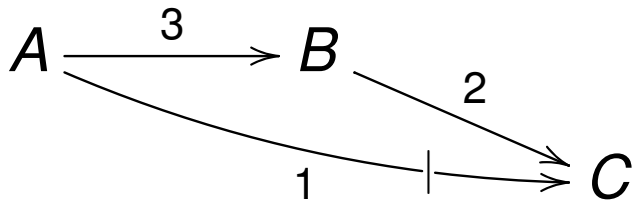
$$\Gamma \preccurlyeq \Phi \equiv (\forall \alpha \in \Gamma \backslash \Phi)(\exists \beta \in \Phi \backslash \Gamma)(\alpha \rhd \beta)$$

A rule $A \rightarrow B$ *dominates* a set of rules Γ if the set of implications $\{\vec{\Gamma}, A \supset B\}$ is incompatible with A.

Admissible priority orders

If α dominates Γ, it is prior to at least one conditional in Γ.

Troubles with preferences



The priority order $1 \rhd 2 \rhd 3$ is not admissible, and choosing $\{2, 3\}$ violates specificity and direct inference wrt 1. But

> *Violation of direct inference wrt a rule α can be seen as an eventual cancelation of α due to rule updates.*

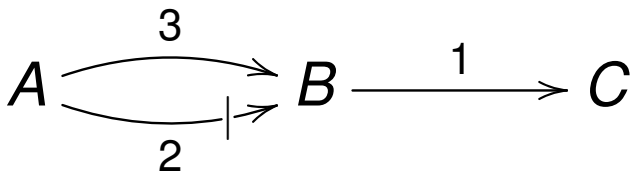
Open problem

What is the right way of combining specificity and prioritization?

Troubles with descriptive preferences

The consensus preference order

$$\Gamma \preccurlyeq \Phi \equiv (\forall \alpha \in \Gamma \backslash \Phi)(\exists \beta \in \Phi \backslash \Gamma)(\alpha \rhd \beta)$$



In the case $3 \rhd 2 \rhd 1$, $\{1, 3\}$ is preferred to $\{2\}$, contrary to

Principle II (Brewka & Eiter)

If Γ is a preferred extension of $(\Delta, \rhd)$, and $\alpha = A \rightarrow B$ is such that $A \notin \mathrm{Th}(\Gamma)$, then Γ is a preferred extension of $(\Delta \cup \{\alpha\}, \rhd_1)$ whenever $\rhd_1$ agrees with $\rhd$ on Δ.

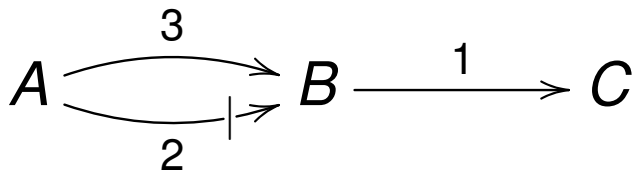
Troubles with prescriptive preference

- *Prescriptive preferences*: Priorities should constrain the order of rule application.

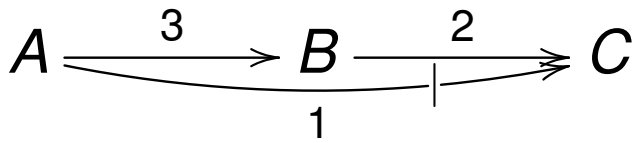
Priority-preserving extensions

Γ is a *p-p extension* if there is a total order $<$ on Δ s.t.

- if $\alpha \rhd \beta$, then $\beta < \alpha$;
- If $\delta = A \rightarrow B \in \Delta \backslash \Gamma$ and $A \in \mathrm{Th}(\Gamma)$, then $\Gamma|_{<\delta}$ defeats δ.



If $3 \rhd 2 \rhd 1$, then $\{2\}$ is the only p-p extension. But

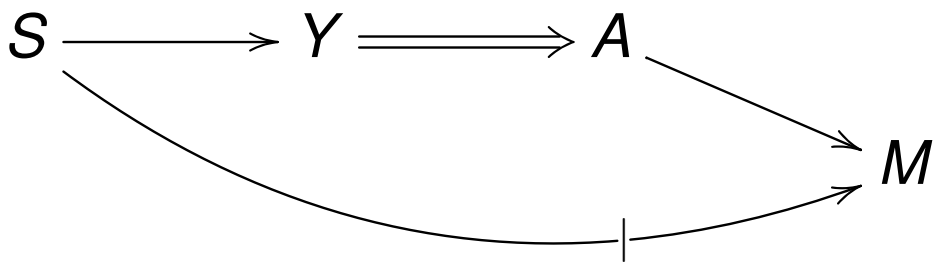


For $3 \rhd 1 \rhd 2$ there is no p-p extension.

Context-Dependent Priorities

Example (Dung and Son, 2001)

1. Students are normally young adults ($S \rightarrow Y$);
2. Young adults are adults ($Y \Rightarrow A$);
3. Adults are normally married ($A \rightarrow M$);
4. Students are normally not married ($S \rightarrow \neg M$).



- Evidence S: should imply $\neg M$; *but*
- Evidence $S \wedge \neg Y \wedge A$ should not decide M: the marital status of non-young students is not determined.

Argumentation and Default Reasoning

- *Default/Argumentation approach is more general and subsumes the preferential approach in reasoning with conditionals.*
- Dung argumentation theory is *abstract*; it is (at best) only an output of a more complex logical theory of assumption-based argumentation [Bondarenko et al].
- The set of fundamental relations among arguments is much richer than an attack relation.
- Default logic is a primary instantiation of an assumption-based argumentation.

Default Approach to Conditionals

Default assumptions of $A \to B$ form a conditional, A/B, that sanctions the inference from A to B:

$$A \to B \equiv A \wedge (A/B) \supset B$$

For any rule $A \to B$,

- The axioms include

  MP $\quad A \wedge (A/B) \supset B.$

- Default rules include

$$A : A/B \vdash A/B.$$

The Logic of Conditionals

An extension of classical propositional language with new atoms A/B, where A and B are classical propositions.

MP $\quad A \wedge (A/B) \supset B.$

The rules of a supraclassical consequence relation:

Dominance $\quad$ If $A \models B$, then A/B.

Transitivity $\quad$ If A/B and B/C, then A/C.

And $\quad$ If A/B and A/C, then $A/(B \wedge C)$.

Forward chaining:

Forward Rejection $\quad \dfrac{A/B \quad \neg(A/C)}{\neg(B/C)}$

Rejection and commitment

The commitment principle: If $A \to B$ is a default, no acceptable combination of defaults could derive $\neg B$ from A.

Commitment

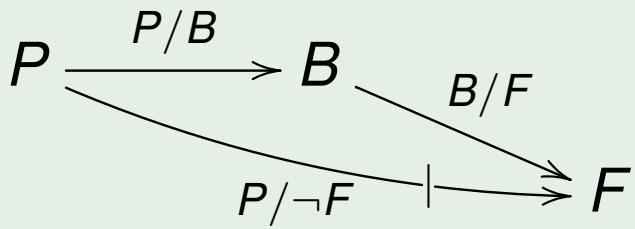
If $A \to B$ is accepted, then

$$A \vdash \neg(A/\neg B).$$

The resulting theory provides a precise formalisation of *nonmonotonic inheritance*.

Example

Example (A generalized Penguin-Bird story)



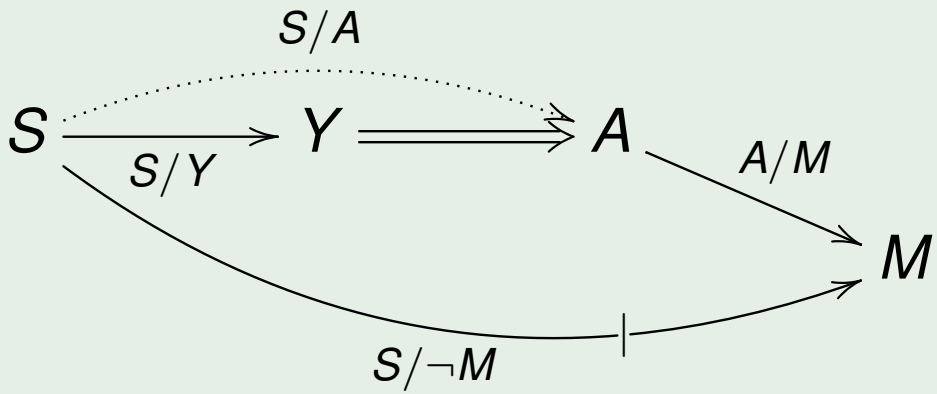
- A unique extension of B that contains F.
- A unique extension of P that includes B and $\neg F$.

The spurious extension of P containing F is blocked by commitment to $P \rightarrow \neg F$ that implies (with Forward Rejection)

$$P,\ P/B \vdash \neg(B/F).$$

Married Students, Revisited

Example



- Evidence S: the only extension contains $\neg M$ and S/A, so $\neg(A/M)$ by commitment to $S \rightarrow \neg M$.
- Evidence $S \wedge \neg Y \wedge A$: refutes S/Y, so two extensions, one containing M, another containing $\neg M$.

Conditionals and Argumentation

Conditionals = (primitive) Arguments

- Attack of rules by facts and other rules:

$$F, \Gamma \vdash \neg(A/B).$$

- Constitutional principles: $\vdash \neg(A/B)$.
- Support: $F, \Gamma \vdash A/B$.

Belief formation and reasoning with conditionals amount to argumentation in a suitable logic.

Main Conclusions

- Evolutionary dynamics of rules is representable via expansions of (prioritized) argumentation systems.
- Argumentation theory should be returned to (and combined with) Logic.