Conditional Acceptance Functions

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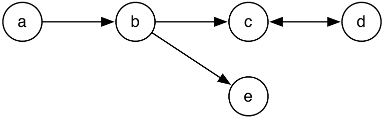
Argumentation Frameworks

Definition

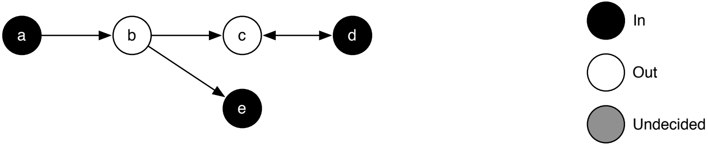
An *argumentation framework* F is a pair (A_F, R_F), where A_F is a set of *arguments*, and $R_F \subseteq A_F \times A_F$ is an *attack relation*. We denote the set of all argumentation frameworks by $\mathcal{F}$.



Labeling

Definition

Given a framework F, a *labeling* is a function $L : A_F \rightarrow V$, where $V = \{I, U, O\}$. We denote the set of all labelings by $\mathbb{L}_F^{all}$.



Acceptance Functions

Definition

An *acceptance function* is a function $\mathbb{A}$ that returns, for any $F \in \mathcal{F}$, a set $\mathbb{A}_F \subseteq \mathbb{L}_F^{all}$.

Definition

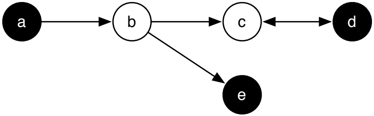
Given a framework F, the *complete* acceptance function $\mathbb{A}_F^{co}$ returns all labelings such that, $\forall a \in A_F$,

- $L(a) = I$ iff $\forall (b, a) \in R_F, L(b) = O$
- $L(a) = O$ iff $\exists (b, a) \in R_F, L(b) = I$

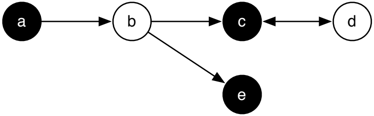
Acceptance Functions

- *Preferred:*
  $\mathbb{A}_F^{pr} = \{L \in \mathbb{A}_F^{co} \mid \nexists K \in \mathbb{A}_F^{co}, K^{-1}(I) \supset L^{-1}(I)\}$
- *Grounded:*
  $\mathbb{A}_F^{gr} = \{L \in \mathbb{A}_F^{co} \mid \nexists K \in \mathbb{A}_F^{co}, K^{-1}(U) \supset L^{-1}(U)\}$
- *Stable:*
  $\mathbb{A}_F^{st} = \{L \in \mathbb{A}_F^{co} \mid L^{-1}(U) = \emptyset\}$

Three complete labelings



Also stable and preferred



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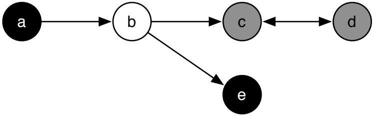


Also grounded

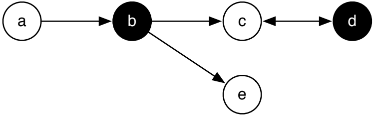
A form of the closed world assumption

- The closed world assumption is the assumption that what is not currently known to be true, is false.
- Here we assume that arguments currently known to be attacked only by OUT labeled arguments, are labeled IN.
  - Or: If something is not falsified, then it is true.

A form of the closed world assumption

- If we view a framework as the theory of an agent, then complete semantics tells the agent what to believe, given that his knowledge is complete.
- This may be appropriate for some applications, but as a theory, an argumentation framework can be used more generally.
  - Persuading another agent, or persuading an audience
  - Counterfactual reasoning
  - Explanation
  - ...

A form of the closed world assumption



Not complete, not admissible

Conditional Acceptance Functions

Definition

A *conditional acceptance function* is a function $\mathbb{CA}_F : 2^{\mathbb{A}_F} \rightarrow 2^{\mathbb{A}_F}$ such that $\mathbb{CA}_F(X) \subseteq X$.

Intuitively, $\mathbb{CA}_F : 2^{\mathbb{A}_F} \rightarrow 2^{\mathbb{A}_F}(X)$ returns those labelings from X that are 'most rational'

Definition

A conditional acceptance function $\mathbb{CA}_F$ *generalizes* an acceptance function $\mathbb{A}_F$ if and only if $\mathbb{CA}_F(\mathbb{L}_F^{all}) = \mathbb{A}_F$.

Conditional Acceptance Functions

Definition

Given a framework F, the *conditionally preferred, grounded and stable* acceptance functions, denoted by $\mathbb{CA}_F^{pr}, \mathbb{CA}_F^{gr}$ and $\mathbb{CA}_F^{st}$, respectively, are defined as follows.

- $\mathbb{CA}_F^{pr}(X) = \{L \in X \cap \mathbb{A}_F^{co} \mid \nexists K \in X, K^{-1}(I) \supset L^{-1}(I)\}$
- $\mathbb{CA}_F^{gr}(X) = \{L \in X \cap \mathbb{A}_F^{co} \mid \nexists K \in X, K^{-1}(U) \supset L^{-1}(U)\}$
- $\mathbb{CA}_F^{st}(X) = \{L \in X \cap \mathbb{A}_F^{co} \mid L^{-1}(U) = \emptyset\}$

Conditional completeness

- What if the input does not contain complete labelings. Which labelings can then be considered most complete?

Conditional completeness

Subcompleteness

A minimal condition we impose is subcompleteness:

Definition

Given a framework F, we say that a labeling L is *subcomplete* iff: if $\forall a \in A$,

- if $L(a) = I$ then for every neighbor b of a, $L(b) = O$, where a neighbor of a is an argument b such that $(a, b) \in R_F$ or $(b, a) \in R_F$.

We denote the set of subcomplete labelings by $\mathbb{L}_F^{sc}$.

Conditional completeness

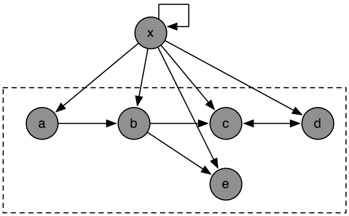
Subcompleteness is motivated by the ‘embeddability property’. Informally:

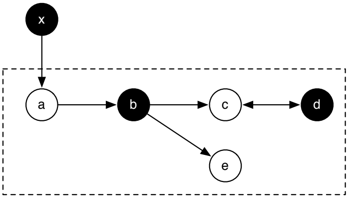
Definition

A labeling of F is *embeddable* if it is part of a complete labeling of some bigger framework G, that extends F with additional arguments and attacks.

Conditional completeness

Embeddability of subcomplete labelings (examples)





Conditional completeness

Given a set of subcomplete labelings X, how do we determine which are 'most complete'?

Definition

Given a framework F and a set $X \subseteq \mathbb{L}_F^{sc}$, we say that a labeling $L \in X$ is *complete given* X iff $\forall a \in A_F$:

1. If $L(a) = U$ then either $(\forall K \in X, K(a) \leq U)$ or $\exists (b, a) \in R_F, L(b) = U$.
2. If $L(a) = O$ then either $(\forall K \in X, K(a) = O)$ or $\exists (b, a) \in R_F, L(b) = I$.

Conditional completeness

Definition

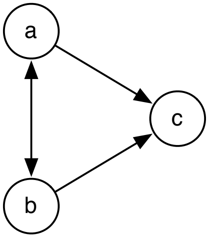
Given a framework F, the *conditionally complete acceptance function* $\mathbb{CA}_F^{co}$ is a conditional acceptance function defined by $\mathbb{CA}_F^{co}(X) = \{L \in X \cap \mathbb{L}_F^{sc} \mid L \text{ is complete given } X \cap \mathbb{L}_F^{sc}\}$.

Conditional completeness

Example (1)

Subcomplete labelings:
($v_1 v_2 v_3$ means $L(a) = v_1, L(b) = v_2, L(c) = v_3$):

OOO	UOO	**IOO**
OOU	UOU	
OOI	UUO	
OUO	**UUU**	
OUU		
OIO		



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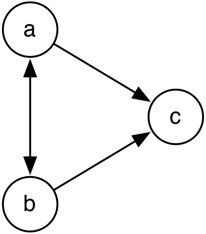

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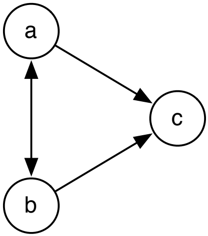

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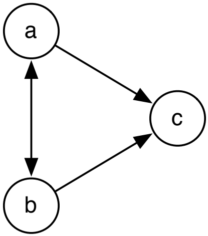

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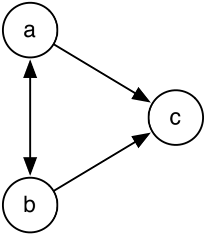


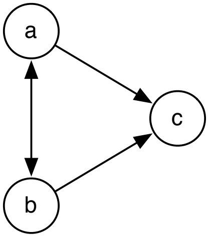
Note: According to directionality, c should not affect a and b. One complete labeling assigns (UUU). But there is no complete labeling (UUO). Limiting ourselves to complete labelings would have destroyed the option of assigning U to a and b, when restricting c to O.

Conditional completeness

Example (2)

Subcomplete labelings:
($v_1 v_2 v_3$ means $L(a) = v_1, L(b) = v_2, L(c) = v_3$):

OOO	UOO	**IOO**
OOU	UOU	
OOI	UUO	
OUO	**UUU**	
OUU		
OIO		



Conditional completeness

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Subcomplete labelings:
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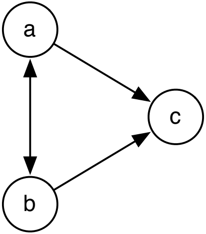
OOU
OOI



Conditional completeness

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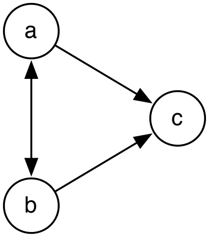
OOI



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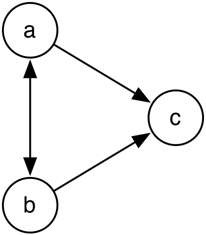

Note: There was no complete labeling assigning I to c.

Conclusions and future work

Conclusions:

- We have generalized the concept of an acceptance function.
- With this generalization, argumentation frameworks can be applied more generally.

Future work:

- Refine our new concepts.
- Try to apply this in an instantiated setting.
- Apply this to models of persuasion dialogs.