A Fundamental Problem of Deontic Logic

Literature:
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- Jörg Hansen: Problems and Results for Logics about Imperatives. JAL 2 (2004), 39-61
- Jörg Hansen: Conflicting Imperatives and Dyadic Deontic Logic, JAL 3 (2005), 1-34
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1. Jörgensen's Dilemma

While normative concepts are the subject of deontic logic, it is quite difficult how there can be a logic of such concepts at all. Norms like individual imperatives, promises, legal statutes, moral standards etc. are usually not viewed as being true or false. E.g. consider imperative or permissive expressions such as “John, leave the room!” and “Mary, you may enter now”: they do not describe, but demand or allow a behavior on the part of John and Mary. Being non-descriptive, they cannot meaningfully be termed true or false. Lacking truth values, these expressions cannot – in the usual sense – be premise or conclusion in an inference, be termed consistent or contradictory, or be compounded by truth-functional operators. Hence, though there certainly exists a logical study of normative expressions and concepts, it seems there cannot be a logic of norms: this is Jörgensen’s dilemma [Jörgen Jörgensen, Imperatives and Logic, Erkenntnis 7, 1938, 288-296].

2. Makinson's Fundamental Problem

Deontic Logic is the study of deontic sentences, i.e. sentences that describe that something (an action, a state of affairs) is obligatory, permitted or forbidden. The truth and falsity of such sentences obviously depends on whether there exist – in the real world – norms that make the state of affairs or action so. To model such truth and falsity, it seems necessary to distinguish, on the semantic level, between norms (that cannot be true or false) and deontic sentences (that are true and false, and are evaluated with respect to the set of norms). In truth, however, the work on deontic logic has been going on as if the distinction between norms and deontic sentences "has never been heard of" (David Makinson 1998):

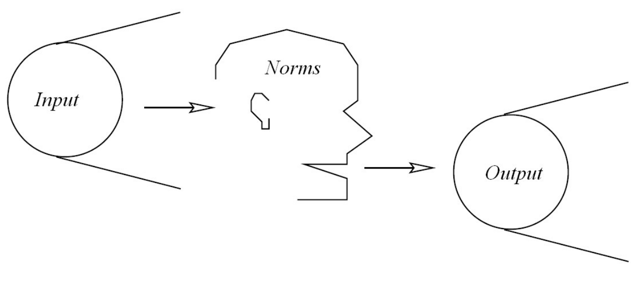
> “In axiomatic presentations of systems of deontic propositional logic, the truth-functional connectives ‘and’, ‘or’ and most spectacularly ‘not’ are routinely applied to items construed as norms, forming compound norms out of elementary ones (...). In semantic presentations, models are constructed that blithely assign truth values to norms in possible worlds, and define validity in a model as truth in all its worlds.”

David Makinson therefore called for a fresh start in deontic logic:

> “It is thus a central problem – we would say: **a fundamental problem** – of deontic logic to **reconstruct it in accord with the philosophical position that norms are devoid of truth values**. In other words: to explain how deontic logic is possible on a positivistic philosophy of norms.”

3. A solution to the Fundamental Problem: Models of norms

The basic idea:



Input/output logic: output = norms
(judicial reasoning; “Da mihi factum, dabo tibi ius”)

Imperative tradition of DL: output = deontic sentences
(reasoning of a moral/legal advisor)

a) Reconstruction of monadic deontic logic

Let I be a set of propositions, they are meant to correspond to the termination statements of a set of (unconditional) imperatives. Then we can e.g. define:

(a) $I \vDash OA$ iff $A \in I$

(b) $I \vDash OA$ iff there is a $B \in I : \vdash_{PL} A \leftrightarrow B$

(c) $I \vDash OA$ iff there is a $B \in I : \vdash_{PL} B \rightarrow A$

(d) $I \vDash OA$ iff for some $B_1, ..., B_n \in I : \{ B_1, ..., B_n \} \vdash_{PL} A$

(e) $I \vDash OA$ iff $I \vdash_{PL} A$

Then: the corresponding 'modal' logics for O are defined by (a) just all tautologies plus modus ponens, (b) plus the rule of extensionality (Ext), (c) plus all instances of axiom (M), (d) plus all instances in axiom (C), (e) plus axiom (N). We obtain SDL from (e) by assuming that I is consistent.

b). Reconstruction of dyadic deontic logic: reasoning in "bad" circumstances

Let I be as before and define

(a) $I \vDash O(A/C)$ iff for all $\Gamma \in I \perp \neg C$: $\Gamma \cup \{C\} \vdash_{PL} A$

(b) $I \vDash O(A/C)$ iff for some $\Gamma \in I \perp \neg C$: $\Gamma \cup \{C\} \vdash_{PL} A$

Then for (a) the corresponding dyadic system is defined by the following axioms and rules (plus modus ponens):

(DM)	$O(A \wedge B/C) \rightarrow (O(A/C) \wedge O(B/C))$
(DC)	$(O(A/C) \wedge O(B/C)) \rightarrow O(A \wedge B/C)$
(DN)	$O(\top/C)$
(DD-R)	If $\nvdash_{PL} \neg C$ then $\vdash_{DDL} O(A/C) \rightarrow P(A/C)$
(Cond)	$O(A /C \wedge D) \rightarrow O(D \rightarrow A /C)$
(CCMon)	$O(A \wedge D /C) \rightarrow O(A /C \wedge D)$
(CExt)	If $\vdash_{PL} C \rightarrow (A \leftrightarrow B)$ is a tautology then $\vdash_{DDL} O(A/C) \leftrightarrow O(B/C)$
(ExtC)	If $\vdash_{PL} C \leftrightarrow D$ is a tautology then $\vdash_{DDL} O(A/C) \leftrightarrow O(A/D)$

For (b) the corresponding dyadic system
- lacks (DC)

- (DN) becomes	(DN-R)	If $\nvdash_{PL} \neg C$ then $\vdash_{DDL} O(\top/C)$
- (DD-R) becomes	(DP)	$P(\top/C)$
- we additionally have	(RMon)	$P(D/C) \rightarrow (O(A /C) \rightarrow O(A /C \wedge D))$

3. How do we reconstruct reasoning about conditional norms?

Proposal: let I be a set of pairs (A,B) and apply input/output logic in the following way:

$I \vDash O(A/C)$ iff $A \in \text{out}(I, \{C\})$

Problem:

- which output operation is the correct one?
- if conditional norms are not just unconditional norms with material implication for their content, then , out2+ and out 4+ will not work
- if "ghostly contraposition" is to be avoided, then we cannot use out4.

Ghostly contraposition: $I=\{(A,B), (\neg A, C)\} \vDash O(B/\neg C)$

- so it seems we have to choose between 'reasoning by cases' and 'reusable output'.